

Greenflation?

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May 2023

Abstract

What are the real and nominal implications of a green transition to a state with sustainable energy production, and how should monetary policy react during such transition? Using a New-Keynesian model with an energy and a goods sector, we show that a green transition requires the relative price of energy to increase and the relative price of goods, the marginal cost of production, and the real wage to fall. We prove analytically that if energy is not used in production and nominal wages and goods prices are rigid, a flexible energy price and a monetary policy rule that sees through energy-price changes are sufficient for replicating the flex-price economy. If energy is used in production there will be deviations from efficiency but because energy's share of income is small, these deviations are marginal unless the increase in the carbon tax is aggressive and/or monetary policy ill suited. During the green transition, it is optimal for monetary policy to see through the increasing energy prices and focus on core inflation. The result is a modest increase in CPI.

Keywords: Inflation, green transition, monetary policy, climate change

JEL: E52; E58; Q43

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†Vestin: Sveriges Riksbank; We are grateful for comments from Lars Svensson, Per Krusell, and seminar participants at the London School of Economics, the Riksbank, and the CEBRA Annual Meeting 2023. The opinions expressed in this article are the sole responsibility of the authors and should not be interpreted as reflecting the views of Sveriges Riksbank or the ECB.

1 Introduction

It is by now, well established that humans affect the climate by emitting greenhouse gases into the atmosphere, and that this contributes to global warming (IPCC, 2021). Carbon dioxide (CO_2) is the most important greenhouse gas that results from human behaviour, and it is generated as a by-product when fossil fuel is burnt. At the heart of the problem lies an externality where it is too cheap—basically free—to emit greenhouse gases into the atmosphere even though these emissions give rise to climate change and climate related costs.

With the intention to reduce their emissions, some countries have introduced a carbon tax whereas the EU has adopted the European Union Emission Trading System (EU ETS), which is a “cap and trade” scheme that limits the amount of emissions that can come from EU. Carbon taxes and quantity restrictions both lead to a higher price on fossil fuel and both policies are efficient in reducing emissions. The EU has taken even further measures with the adoption of the “Fit-for 55” package that aims at reducing net greenhouse gas emissions by at least 55% (relative to the 1990 level) by 2030. This reduction is achieved by linearly reducing the newly emitted emission rights—and thus increasing the price—each year going forward.

In the central banking community there is some concern for “greenflation”, which we here define broadly to refer to as the upward pressures on inflation that might arise from a green transition to a state with sustainable energy production.¹ It seems plausible that the concern for greenflation has been amplified by the recent experience with surging inflation, which at least partly has been driven by supply shocks to the energy sector.

Against this background, we first assess the real and inflationary implications that the green transition will bring about. We then proceed by determining how monetary policy should react during the transition.

To study these issues, we set up a New-Keynesian model with two sectors: one that combines fossil and green energy sources as inputs to produce energy services, and one that combines labor and energy services to produce consumption goods. Households consume both goods and energy services. Wages and prices of consumption goods are subject to nominal frictions and therefore sticky but we assume the energy price to be fully flexible, which seems reasonable at a quarterly time horizon.

The model economy is thought of as a region such as the European Union that embarks

¹See, for instance, the speech by Isabel Schnabel, March 17, 2022. Schnabel makes a distinction between “fossilflation” and “greenflation”. The former term refers to the higher cost of carbon as well as to high fossil fuel prices, whereas the latter term refers to the higher cost associated with high green-energy-production costs. We do not make a distinction between these two concepts, but use the “greenflation” more broadly to incorporate all potential upward pressures on costs and prices that would be associated with the green transition.

on a green transition by implementing a policy that increases the price of fossil energy so that fossil energy use falls by about 60% in the coming decades (similar to the Fit for 55 package). We focus on a carbon tax but the results would, in our setting, be identical with a cap-and-trade system. Finally, as a benchmark we assume that no policy to reduce emissions is introduced in the rest of the world. Because the share of emission that are coming from the EU only accounts for about 10 % of global emissions, the effect of the EU policy on the global externality is ignored.²

We also contrast the benchmark setting with sticky wages and goods prices to one with fully flexible prices and wages. Because the transition in the flex-price economy is free from market failures, it provides the least costly way to achieve a specific desired reduction in domestic fossil fuel use. For that reason, we will refer to the flex-price allocation as the *the efficient green transition*.³ Nominal and/or other frictions can then only make the transition more costly.

Turning to the results, we first show analytically that the efficient green transition in a relatively small region requires the relative price of energy to increase and the relative price of goods, the marginal cost of producing goods, the real wage, output, and consumption to fall in that region.⁴ The inflationary consequences of these adjustments then depend critically on how monetary policy is conducted and the extent to which the economy features sticky prices and wages. If all prices are flexible, the necessary adjustments can come about without any consequences for inflation. For instance, if the central bank follows a Taylor rule that features a enough high weight on stabilizing inflation around the inflation target, the price of goods will fall as the energy price increases and headline inflation will be zero.⁵ In the flex-price setting this has no cost, since nominal variables have no real effects. With CPI targeting and a lower weight on inflation stabilization, there will instead be slight *deflation* from the green transition.⁶

We then prove analytically in our framework, that if energy is not used in production and nominal wages and goods prices are so rigid that they cannot adjust at all, a flexible energy price and a monetary policy rule that sees through energy-price changes are sufficient for implementing the efficient green transition. In other words, the increase in CPI

²This is a quantitatively realistic assumption as shown in Hassler, Krusell, and Olovsson (2020).

³Note that the desired reduction is important here. Otherwise, efficiency from a regional perspective would require a zero carbon tax in all periods.

⁴The fall in real income and consumption is due to the assumption that the only market failure in the flex-price model comes from the emission of carbon dioxide and because the carbon tax in a single region cannot eliminate the externality, the tax is simply distortionary for the region.

⁵The result that if prices are flexible then the green transition do not need to have any consequences for inflation was simultaneously independently derived in a somewhat different model in Del Negro et al. (2023).

⁶This follows from that consumption is falling during the transition, which implies a falling real rate. We show that inflation is a product of all future discounted real interest rates, where these rates being below or equal to the pre-transitional values.

that is generated by the increase in the energy price is exactly the increase that is needed to lower the real wage and the real goods price to their efficient levels. The reason for this potentially surprising result is that if energy is not used in production, the wage and the price of goods are equal (both in real and nominal terms). Hence, when the energy price adjusts so that the relative price between energy and labor is efficient, then also the relative price between energy and goods will be efficient.

In the more realistic setting where nominal goods prices and wages are not fully fixed but sticky, these prices can potentially move during the transition. With nominal frictions such movements are associated with costs. The efficient monetary policy is then to make sure that either core or wage inflation remains on target, while seeing through the increase in the energy price. The resulting increase in the CPI from this policy is modest: the maximum yearly inflation rate peaks well below one percent. More importantly, this inflation is not costly since the efficient green transition is implemented.

If energy is used in production, however, it is no longer sufficient with a flexible energy price and a monetary policy rule that sees through energy-price changes to implement the efficient green transition when nominal wages and goods prices are sticky or cannot adjust. The price of energy can then either adjust so that the relative price between energy and non-energy goods is correct, or so that the relative price between energy and wages is correct but not so that both these relative prices are correct. Hence, either the nominal wage, the nominal price of goods or a combination of both will have to make costly adjustments.

For this case, we show analytically that the deviations from the efficient green transition depends critically on two parameters: the weight on energy in production and the magnitude of the carbon tax. The weight on energy in production is, by any estimate, small: energy's share of income is between 0.02 and 0.05. As a result, the deviations of the real wage and the real price of goods from their respective efficient levels will also be small even in the presence of nominal frictions.

The only way that deviations could be non-trivial would be if the change in the carbon tax would be large, i.e., like a shock to the energy price. However, if the carbon tax is phased in as is the case with the Fit for 55 package, deviations from efficiency will, at each point in time, be limited.

We then verify numerically that an economy with energy in production, sticky wages and goods prices, and a monetary policy rule that prevents wage inflation from deviating from target but allows the goods price to adjust comes very close to replicating the efficient transition. The median deviation in consumption from the efficient level is only 0.002 percent during the transition and the maximum deviation is 0.02. The resulting increase in the CPI is again modest and peaks below one percent.

We also show that if either wages or goods prices are flexible—in addition to the energy price—then it is always possible to replicate the efficient green transition with a simple Taylor rule even when energy is used as an input in production. If wages are sticky and goods prices flexible, the optimal monetary policy response is to prevent wage inflation from deviating from target. If instead, goods prices are sticky and wages flexible, the response is to prevent goods inflation from deviating from target. In both cases, the efficient green transition is replicated and there is a mild (non-costly) increase in CPI.

Finally, we verify our analytical finding that a drastic and immediate introduction of the carbon tax can potentially generate relatively high and immediate inflation. Specifically, a transition that aggressively increases the carbon tax in an economy where prices and wages are sticky does, indeed, induce relatively large deviations from efficiency prices and wages.

It should be pointed out that we employ simple Taylor rules to illustrate that it is possible to either replicate the flex-price economy or to get very close. In some cases these simple rules will have extreme parametrizations, but these rules should just be seen as illustrations of what is possible. Using optimal policy, it would be possible to design more sophisticated rules that also allow for additional shocks.

We conclude that during the green transition, monetary policy should see through the increasing energy prices and instead focus on core or wage inflation. With this policy, a green transformation of the economy can come about with only modest inflationary pressures or, so called, greenflation. These theoretical results are in line with the empirical findings in Konradt and Weder di Mauro (2023), showing only limited effects on inflation from green policies.⁷

The outline of the paper is as follows. Section 2 introduces the main model and Section 3 outlines the calibration of the model. Section 4 presents the results, starting with the analytic results and then moves on to the numerical results. A general discussion about features omitted from the model such as optimal monetary policy, green subsidies instead of carbon taxes, endogenous capital, and climate change is found in Section 5 and Section 6, finally, concludes.

2 A model with sticky wages and prices

We now set up a relatively standard New-Keynesian model with sticky prices and wages, but extended with an energy sector. The basic framework builds upon Woodford (2003) and Gali (2015), whereas the energy sector is taken from Hassler, Krusell, and Olovsson

⁷Airaudo et al. (2022) finds a relatively strong impact on inflation in the beginning of the transition, which could come from the fact that there is no announcement before the transition starts.

(2020, 2021a). In an extension, we show how to introduce climate change into the model.

We start by specifying the full model. By only changing a few assumptions, however, the model collapses to a RBC model with flexible nominal prices and wages—a model that can be solved in closed form. Due to the absence of nominal rigidities, inflation and monetary policy then have no implications for the real variables and welfare. However, the model serves as an important benchmark, since it reveals the least costly way to achieve a specific desired reduction in domestic fossil fuel use. The flex-price model is also used to derive two propositions that constitute two of the most important results in the paper. Finally, it is very useful for providing intuition.

2.1 Households

We consider an economy with a continuum of labor-types indexed by j .⁸ The different types are organized in unions that sets wages. Given the wage, the respective types have to satisfy demand, which implies that the number of hours worked will generally not be optimal for each of the types.⁹ We further assume complete markets on the consumer side, which allows the income risk to be perfectly shared and consumption levels to be equalized. But the number of hours worked will not be identical and hence we will see fluctuations in the marginal disutility of labor. The agent derives utility from consumption, C , and disutility from providing labor. Denoting the amount of hours worked of a type j by $H_t(j)$, preferences for the household are given by

$$\sum_{t=0}^{\infty} \beta^t \log(C_t) - \psi \int_0^1 \frac{H_t(j)^{1+\varphi}}{1+\varphi} dj \quad (1)$$

where β is the discount factor, ψ and φ respectively determines the disutility of labor and the Frisch elasticity. Consumption is then an aggregate of a manufactured good—that is labelled C_g —and energy services—labelled C_e . Formally, we have

$$C_t = \left((1 - \nu)^{\frac{1}{\eta}} C_{g,t}^{\frac{\eta-1}{\eta}} + \nu^{\frac{1}{\eta}} C_{e,t}^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}. \quad (2)$$

where η is the elasticity of substitution between the C_g and C_e . The price of the manufactured and the energy good is respectively denoted by $P_{g,t}$ and $P_{e,t}$, whereas the price of the aggregate consumption basket, C , is P . The budget constraint for the consumer

⁸Our modelling of sticky wages follows Erceg, Henderson, and Levin (2000).

⁹Just like the demand-determined output of the firm is sub-optimal in a given period when the price is fixed.

is then given by

$$P_t C_t + B_t = \int_0^1 W_t(j) H_t(j) dj + \pi_t + I_{t-1} B_{t-1} + T_t, \quad (3)$$

where π_t is profits, W is the nominal wage rate, I_{t-1} is the gross interest rate paid in period t on funds B_{t-1} invested at time $t - 1$, and T denotes the nominal tax revenues that are rebated to the consumer.

2.2 Firms

Labor is used for three activities: the production of goods, fossil fuel, E_1 , and green energy, E_2 , where we use subscript 1 and 2 to respectively denote the fossil and the green sector. Because we want to allow for sticky wages, we assume that the labor employed by firm i consists of a of an aggregate (bundle) of the different types of labor. The total labor supply in the economy is then given by

$$H_t = \left(\int_0^1 H_t(j)^{\frac{\xi_w - 1}{\xi_w}} dj \right)^{\frac{\xi_w}{\xi_w - 1}},$$

where $H_t(j)$ refers to the amount of hours worked by labor type j , and ξ_w is the elasticity of substitution between different labor types. It is straightforward to show that cost minimization by the firms will imply that the demand for the different types will satisfy

$$H_t(j) = \left(\frac{W_t(j)}{W_t} \right)^{-\xi_w} H_t,$$

where $W_t \equiv \left[\int_0^1 W_t(j)^{1-\xi_w} dj \right]^{\frac{1}{1-\xi_w}}$ is the aggregate wage.

Energy services, $C_{e,t}$, are then produced by a competitive representative firm that combines the two energy sources as inputs, but this activity does not require any labor. We assume, the prices of energy inputs and energy services to be fully flexible—which is reasonable for a quarterly horizon—but the price of the non-energy good, P_g , is sticky.

2.2.1 The goods sector

Sector g is assumed to consist of a continuum of price setting firms that are optimizing under monopolistic competition. The price of individual good i is then $P_t(i)$. The production function for goods is a CES aggregator in labour and energy, i.e.,

$$Y_{g,t}(i) = \left[(1 - \gamma) (A_{g,t} H_{g,t}(i))^{\frac{\rho-1}{\rho}} + \gamma (e_{g,t}(i))^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}}, \quad (4)$$

where $H_{g,t}(i)$ and $e_{g,t}(i)$ respectively is the labor and energy demanded by firm i (not to be confused with that is the labor supplied by labor type j), and ρ is the elasticity of substitution between labor and energy. We will also consider the special case where energy is *not* used as an input into production, which amounts to setting $\gamma = 0$.

There also exists a perfectly competitive firm that buys the individual goods and package them to produce a manufactured final good, using the following aggregator

$$Y_{g,t} = \left(\int_0^1 \left(Y_{g,t}(i) \right)^{\frac{\xi-1}{\xi}} di \right)^{\frac{\xi}{\xi-1}}. \quad (5)$$

The cost minimization problem for the packaging firm delivers the individual demand functions for goods of type i , i.e.,

$$Y_{g,t}(i) = \left(\frac{P_{g,t}(i)}{P_{g,t}} \right)^{-\xi} Y_{g,t}, \quad (6)$$

as well as the price index for good g :

$$P_{g,t} = \left(\int_0^1 \left(P_{g,t}(i) \right)^{1-\xi} di \right)^{\frac{1}{1-\xi}}. \quad (7)$$

We assume Calvo pricing, which implies that the probability that a firm is allowed to change its price in each period is $1 - \theta$. Since all firms that are allowed to change their price, will choose the same price, the maximization problem for firm i in period t is given by

$$\max_{P_{g,t}^*} \mathbb{E}_t \sum_{j=0}^{\infty} \theta^j Q_{t,t+j} \left(P_{g,t}^* Y_{g,t+j}(i) - TC_{t+j}(Y_{g,t+j}(i)) \right), \quad (8)$$

s.t.

$$Y_{g,t+j}(i) = \left(\frac{P_{g,t}^*}{P_{g,t+j}} \right)^{-\xi} Y_{g,t+j}, \quad (9)$$

and where TC_{t+j} is the total cost of production in period $t + j$.

2.2.2 Energy production and supply

The energy sector features two types of firms: those that produce the different energy inputs and the energy-service provider that buys these inputs and use them as inputs to produce and supply energy services, $C_{e,t}$ and $e_{g,t}$. The energy inputs are fossil fuel (E_1) and green energy (E_2) and they are both produced using linear technologies. Prices of energy inputs and energy services are assumed to be fully flexible, and only fossil-fuel production is subject to taxation. The profit-maximization problem in the fossil and the

green sector is then, respectively, given by

$$\pi_{1,t} = \max_{H_{e,t}} (1 - \tau_t) P_{1,t} \underbrace{A_{1,t} H_{1,t}}_{E_{1,t}} - W_t H_{1,t}, \quad (10)$$

and

$$\pi_{2,t} = \max_{H_{e,t}} P_{2,t} \underbrace{A_{2,t} H_{2,t}}_{E_{2,t}} - W_t H_{2,t}, \quad (11)$$

where τ denotes the tax on fossil fuel production (and $H_{1,t}$ and $H_{2,t}$ respectively, denote the labor demand by energy producer 1 and 2).

Energy services are then produced by a competitive representative firm that combines the two energy sources as inputs using the following aggregator:

$$C_{e,t} + e_{g,t} = \left((1 - \lambda)^{\frac{1}{\varepsilon}} E_{1,t}^{\frac{\varepsilon-1}{\varepsilon}} + \lambda^{\frac{1}{\varepsilon}} E_{2,t}^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}}. \quad (12)$$

where ε is the elasticity of substitution between the two energy inputs. The cost-minimization problem for the energy service provider is given by

$$\min_{E_{1,t}, E_{2,t}} P_{1,t} E_{1,t} + P_{2,t} E_{2,t} - P_{e,t} \left[C_{e,t} + e_{g,t} - \left((1 - \lambda)^{\frac{1}{\varepsilon}} E_{1,t}^{\frac{\varepsilon-1}{\varepsilon}} + \lambda^{\frac{1}{\varepsilon}} E_{2,t}^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}} \right]. \quad (13)$$

2.3 Unions

The union is setting the wage that maximizes the expected utility of its members. Since the wage only is allowed to change with a certain probability, the intertemporal problem becomes

$$\max_{W_t^*} \mathbb{E}_t \sum_{k=0}^{\infty} (\beta \theta_w)^k \left(\log(C_{t+k}) - \psi \int_0^1 \frac{H_{t+k|t}(j)^{1+\varphi}}{1+\varphi} dj \right) \quad (14)$$

s.t.

$$P_{t+k} C_{t+k} + B_{t+k} = \int_0^1 W_t^*(j) H_{t+k}(j) dj + \pi_{t+k} + I_{t+k-1} B_{t+k-1} + T_{t+k},$$

and

$$H_{t+k|t}(j) = \left(\frac{W_t^*}{W_{t+k}} \right)^{-\xi_w} H_{t+k},$$

and where $1 - \theta_w$ is the probability that the wage for type j can be changed in each period.

2.4 Monetary policy

We start by considering a monetary policy rule of the form

$$I_t = \beta^{-1} \Pi^* \left(\frac{\Pi_t}{\Pi^*} \right)^{\alpha_\pi}, \quad (15)$$

where Π^* is the gross inflation target and

$$\Pi_t \equiv \frac{P_t}{P_{t-1}} \quad (16)$$

is the inflation rate. We will not focus exclusively on CPI as the only inflation target but we will also consider alternative targets such as core and wage inflation. As we will see below, these types of simple Taylor-rule—without any response to the output-gap—can go a long way in obtaining efficiency.

2.5 Climate change

In the benchmark scenario, we do not allow either fiscal or monetary policy to have any effect of the climate. The reason is that we are considering policy changes in a small region, such as the EU that only has a marginal effect on global emissions.¹⁰ The share of global emissions that are coming from the EU is about 10 percent and that share is expected to be reduced in the coming decades. In an extension, we show how the model easily can be modified to allow for an interaction between the economy and the climate.

2.6 Equilibrium

We now describe the equilibrium conditions.

2.6.1 Households

We will use the following definitions $p_{g,t} \equiv P_{g,t}/P_t$, $p_{e,t} \equiv P_{e,t}/P_t$, $w_t \equiv W_t/P_t$, $b_t \equiv B_t/P_t$. All derivations of the optimality conditions are laid out in detail in the Appendix. The

¹⁰See Hassler, Krusell, and Olovsson (2020).

maximization problem for the consumer can be written as follows.

$$C_{g,t} = (1 - \nu) p_{g,t}^{-\eta} C_t, \quad (17)$$

$$C_{e,t} = \nu p_{e,t}^{-\eta} C_t, \quad (18)$$

$$\frac{1}{C_t} = \beta \mathbb{E}_t \left(\frac{1}{C_{t+1}} \frac{I_t}{\Pi_{t+1}} \right), \quad (19)$$

$$1 = [(1 - \nu) p_{g,t}^{1-\eta} + \nu p_{e,t}^{1-\eta}]^{\frac{1}{1-\eta}}. \quad (20)$$

where (19) is the Euler equation and (20) gives the aggregate price level, i.e., the CPI. The optimality conditions for fossil and green fuel are respectively given by

$$E_{1,t} = (1 - \lambda) \left(\frac{A_{2,t}}{A_{1,t}} \frac{1}{1 - \tau_t} \frac{1}{h_t} \right)^{-\varepsilon} (C_{e,t} + e_{g,t}), \quad (21)$$

and

$$E_{2,t} = \lambda \left(\frac{1}{h_{1,t}} \right)^{-\varepsilon} (C_{e,t} + e_{g,t}), \quad (22)$$

where $h_t \equiv \left[(1 - \lambda) \left(\frac{A_{2,t}}{A_{1,t}} \frac{1}{1 - \tau_t} \right)^{1-\varepsilon} + \lambda \right]^{\frac{1}{1-\varepsilon}}$.

2.6.2 Firms

The first order condition to (8) can, after some manipulation, be written as

$$p_{g,t} = \frac{\xi}{\xi - 1} \left[\frac{1 - \theta \Pi_{g,t}^{\xi-1}}{1 - \theta} \right]^{\frac{1}{\xi-1}} \frac{N_{2,t}}{N_{1,t}}, \quad (23)$$

where $N_{1,t}$ and $N_{2,t}$ are laid out in the Appendix.

The energy demand by the firms are

$$e_{g,t} = \gamma D_{g,t} \left(\frac{p_{e,t}}{m c_{g,t}} \right)^{-\rho} Y_{g,t}, \quad (24)$$

where the marginal cost, $m c_g$, for firms in sector g is given by

$$m c_{g,t} = \left[(1 - \gamma) \left(\frac{w_t}{A_{g,t}} \right)^{1-\rho} + \gamma p_{e,t}^{1-\rho} \right]^{\frac{1}{1-\rho}}. \quad (25)$$

2.6.3 Unions

The solution to the union's problem (14) is

$$w_t^* = \left[\frac{\xi_w}{\xi_w - 1} \frac{M_{1,t}}{M_{2,t}} \right]^{\frac{1}{1+\xi_w\varphi}}, \quad (26)$$

where $M_{1,t}$ and $M_{2,t}$ have been placed in the Appendix. The real wage is then given by

$$w_t = \left[(1 - \theta_w) (w_t^*)^{1-\xi_w} + \theta_w \Pi_t^{\xi_w-1} (w_{t-1})^{1-\xi_w} \right]^{\frac{1}{1-\xi_w}}. \quad (27)$$

2.6.4 Market clearing and prices

Labor market clearing requires that the demand for labor equals the supply, i.e.,

$$H_{g,t} + H_{1,t} + H_{2,t} = H_t. \quad (28)$$

Using the first order condition to (11) in combination with (A-12), the marginal cost of labor, w_t , has to equal the marginal product of labor in the green sector, i.e.,

$$w_t = p_{e,t} \frac{A_{2,t}}{h_t}. \quad (29)$$

The budget constraint written in real terms becomes

$$C_t = w_t H_t + \hat{\pi}_{g,t} + \frac{A_{2,t}}{h_t} p_{e,t} H_{1,t} \frac{\tau_t}{1 - \tau_t}, \quad (30)$$

and $\hat{\pi}_{g,t} = p_{g,t} Y_{g,t} - w_t H_g - p_e e_{g,t}$. The supply of goods, fossil and green fuel must all equal the demand for these goods. We have

$$\underbrace{\frac{A_{g,t} H_{g,t}}{(1 - \gamma) D_{g,t} \left(\frac{w_t}{A_{g,t} m_{c,t}} \right)^{-\rho}}}_{Y_{g,t}} = C_{g,t}, \quad (31)$$

$$\underbrace{A_{1,t} H_{1,t}}_{Y_{1,t}} = E_{1,t}, \quad (32)$$

and

$$\underbrace{A_{2,t} H_{2,t}}_{Y_{2,t}} = E_{2,t}, \quad (33)$$

The equations that define the equilibrium are (15), (17)–(33), (A-14)–(A-20) and the unknowns are $H_{g,t}$, $H_{1,t}$, $H_{2,t}$, H_t , C_t , $C_{g,t}$, $C_{e,t}$, $E_{1,t}$, $E_{2,t}$, $e_{g,t}$, $m_{c,g,t}$, b_t , $N_{1,t}$, $N_{2,t}$, $M_{1,t}$,

$M_{2,t}$, $D_{g,t}$, $p_{g,t}$, $p_{e,t}$, Π_t , $\Pi_{g,t}$, $\Pi_{e,t}$, I_t , w_t^* , and w_t .¹¹

3 Calibration

The model is now calibrated. A period is one quarter. There are five preference parameters: β , ψ , φ , ν , and η . The discount factor β is set to 0.995 to generate a real steady-state annual interest rate of about two percent. Parameter ψ determines the disutility of labor and it is set to 2. We then set $\varphi = 3$ to generate a Frisch elasticity of about 0.5. The weight on energy in the consumption bundle, ν , is set to 0.05 to match that about five percent of the total income is spent on energy. The parameter η is the elasticity of substitution (EOS) between energy services and other goods and we set it to 0.25 to match the low substitution elasticity found in Hassler, Krusell, and Olovsson (2021a).

Parameter ε determines the elasticity of substitution between fossil and green energy in the energy bundle, and it is set to 1.5 in line with the values reported in Hassler, Krusell, and Olovsson (2023). The weight on green energy in the CES aggregator, λ is then set to 0.36, which is based on Hassler, Krusell, and Olovsson (2021b).

Regarding price and wage rigidities, we follow Gali (2015) and set $\theta = 0.75$ for the case with sticky prices and $\theta_w = 0.75$ for sticky wages. We also consider cases where θ and θ_w can be zero. Following Gali (2015), the EOS between firms' output varieties, ξ , is set to 6. As is common in the literature, we impose a subsidy on goods production with a negative tax (denoted by τ_g) that exactly offsets the distortion generated by market power. The EOS between labor types, ξ_w , is set to 2.

Turning now to production parameters. In the benchmark scenarios, we abstract from technical change and set $A_{g,t} = A_{1,t} = A_{2,t} = 1$. However, in Section 4.2.4 we consider a scenario with faster green technical change and allow A_2 to grow by 2% per year for a number of periods in the transition.

As already mentioned, climate change is abstracted from. The implication is that the considered region cannot improve welfare by increasing the carbon tax, which is quantitatively correct when the region is taken to be the EU.¹² Finally, for monetary policy we consider several different values for α_{pi} , as well as several different targets. The benchmark calibration is summarized in Table 1.

¹¹The aggregate price level is indeterminate, but the change in the level is determined.

¹²See Hassler, Krusell, and Olovsson (2020) that shows that the global laissez fair is basically quantitatively the same when only the EU implements a carbon tax.

Table 1: Calibration

PARAMETER	VALUE	TARGET
β	0.995	Discount factor
γ	0.05	Weight on energy in production
ε	1.5	EOS between green and brown energy
η	0.25	EOS between C_g and C_e in consumption
λ	0.36	Weight on green energy in production
ψ	2	Disutility of labor
φ	3	Frisch elasticity
ν	0.05	Weight on energy in consumption
ρ	0.15	EOS between e_g and H in production
θ	{0, 0.75}	Calvo prob for prices
θ_w	{0, 0.75}	Calvo prob for wages
ξ	6	EOS between different types of goods
ξ_w	2	EOS between different types of labor
α_π	{1.5, 500}	Taylor rules

3.1 The considered experiments

As described in the Introduction, we think of the region as the European Union and consider a green transition that will result from a fiscal policy that loosely resembles the Fit for 55 policy proposal. Specifically, the EU runs the European Union Emission Trading System (EU ETS), which is a “cap and trade” scheme that limits the amount of emissions that can come from EU. The Fit-for 55 package then aims at reducing net greenhouse gas emissions by at least 55% by 2030 relative to the 1990 level. This will be achieved by linearly reducing the newly emitted emission rights—and thus increasing the price—each year going forward. We will instead consider a carbon tax that linearly increases up to 2040, after which it remains constant. A carbon tax is, in all important aspects that we are concerned with, virtually identical to a cap and trade system. The alternative policy with green subsidies is discussed in Section 5.4,

We consider an experiment where the tax on energy initially is zero and where the expectations of future energy taxes also are anchored at zero. At time t , agents are then informed that a tax path for brown energy will be introduced. The entire path is fully revealed and agents takes for granted that this will not ever be changed again. The benchmark scenario features a transition that is announced five years in advance of the first tax increase. This is similar to the gradual approach of the Fit-for-55 package and it gives the agents some time to adjust. As a comparison, however, we also consider a case where the tax is increased immediately when the transition is announced. The non-linear perfect foresight equilibrium is solved for with the “perfect foresight solver” in Dynare, which implies that the transition path is not linearised.

The rest of the world is assumed to not implement any policies that provide incentives to reduce their emissions. At the time that this paper is written, this seems to be the

most accurate characterization of the current state of the world.

The reason for considering this specific fiscal policy rather than an optimal one is that we are interested in the effects of a policy that actually is considered to be implemented, rather than any theoretical constructs. Global optimality would also require all countries to implement the same carbon tax. In this regard, the Fit for 55 package is likely not too far from optimal if it were to be implemented globally.¹³

It is, however, important to note that a green transition that is coming from a policy that reduces greenhouse gas emissions will have real effects. Because the externality is global, a tax or quantity restriction in just one relatively small region such as the EU will only have a marginal effect on the externality, but the policy will increase the real costs of producing and consuming fossil energy in that region.¹⁴ This implies that the climate policy will reduce consumption and output in the implementing region. Only if all (or most of the) regions would implement the carbon tax could consumption and output increase. These implications are important for trends for consumption and output as we will see in the next section.

4 Results

We are now ready to present the results and we start with an economy without nominal frictions. This setting reveals the real changes that needs to come about. With fully flexible prices and wages monetary policy has no allocative effects, but the setting allows us to study both the real and the nominal effects of the green transition. As also mentioned in the Introduction, the flex-price economy provides the least costly way of achieving a specific desired reduction in domestic fossil fuel use, which is why we refer to this allocation as “the efficient green transition”. Nominal and/or other frictions can then only make the transition more costly.

4.1 Flexible prices: the efficient green transition

To remove the nominal frictions from the model, we simply assume that there are no frictions for setting prices and wages, that all labor types and firms are identical (so that they are perfect substitutes), and that also all individual goods in sector g are identical. In this case, the utility function becomes $\sum_{t=0}^{\infty} \beta^t \log(C_t) - \varphi H_t^{1+\varphi} / (1 + \varphi)$ and the production function for goods becomes $Y_{g,t} = \left[(1 - \gamma) (A_{g,t} H_{g,t})^{\frac{\rho-1}{\rho}} + \gamma (e_{g,t})^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}}$. As

¹³Note however that if all regions choose to implement the globally optimal policy then a free-riding problem is still present, i.e., it would then be individually optimal for each region to not implement the policy.

¹⁴See Hassler, Krusell, and Olovsson (2020).

a result, the model collapses to a RBC model with flexible prices and wages that can be solved in closed form.¹⁵ We will, below, only present a few selected equilibrium equations that are relevant for our results, but the full solution is presented in Appendix A.6. Let us now start with the real side of the economy.

4.1.1 Relative prices

With $A_g = A_1 = A_2$, the real wage is given by.

$$w_t = \left[(1 - \nu) \left(1 - \gamma + \gamma \left[(1 - \lambda) \left(\frac{1}{1 - \tau_t} \right)^{1 - \varepsilon} + \lambda \right]^{\frac{1 - \rho}{1 - \varepsilon}} \right)^{\frac{1 - \eta}{1 - \rho}} \right. \\ \left. + \nu \left[(1 - \lambda) \left(\frac{1}{1 - \tau_t} \right)^{1 - \varepsilon} + \lambda \right]^{\frac{1 - \eta}{1 - \varepsilon}} \right]^{\frac{1}{\eta - 1}} \quad (34)$$

It is straightforward to verify that the real wage, w_t , is decreasing in the tax rate for all weights $\nu \in (0, 1)$, $\gamma \in (0, 1)$, $\lambda \in (0, 1)$, and all elasticities $0 \leq \rho < \infty$, $0 \leq \varepsilon < \infty$, $0 \leq \eta < \infty$. As mentioned in the previous section, the only market failure in the flex-price model comes from the emission of carbon dioxide and because the carbon tax in a single region cannot eliminate the global externality, the tax is simply distortionary for the region. Real income thus falls.

Turning to the the relative prices of energy and goods, it is clear that the carbon tax increases the relative price of energy and reduces the relative price of goods. Equations (35) and (36) provide closed form solutions of these prices:

$$p_{g,t} = \underbrace{w_t \left(1 - \gamma + \gamma \left[(1 - \lambda) \left(\frac{1}{1 - \tau_t} \right)^{1 - \varepsilon} + \lambda \right]^{\frac{1 - \rho}{1 - \varepsilon}} \right)^{\frac{1}{1 - \rho}}}_{mc_{g,t}} \quad (35)$$

$$p_{e,t} = w_t \left[(1 - \lambda) \left(\frac{1}{1 - \tau_t} \right)^{1 - \varepsilon} + \lambda \right]^{\frac{1}{1 - \varepsilon}}, \quad (36)$$

where w_t is given by (34). Because the model features a tax that completely offsets the distortion generated by market power, p_g always exactly equals the marginal cost of producing goods, mc_g .

A reduction of $p_{g,t}$ requires the l.h.s. of (35) to fall. Consider, now, the different implications of when energy is used as an input in production ($\gamma > 0$) relative to when

¹⁵The price level is indeterminate but the inflation rates are not.

it is not ($\gamma = 0$). In the former case, the carbon tax increases the r.h.s. of (35).¹⁶ It is the reduction in the real wage rate that then ensures that (35) still holds.¹⁷ The real wage will thus have to fall more than the relative price of goods to offset the upward pressure from the carbon tax on the marginal cost of producing goods.¹⁸ The fact that the real marginal cost of production of goods falls when the price of energy increases is potentially surprising, but it is a general equilibrium effect that comes from the required reduction in the real wage.

The falling real wage also puts downward pressure on the energy price, but here the direct effect from the higher carbon tax dominates, i.e., the energy price is increasing in the tax rate.

Note now that if energy is not used in production, then (35) collapses to $p_{g,t} = w_t$, which also implies $P_{g,t} = W_t$. Hence, the price of goods and the wage rate will then always move exactly one for one. The implication of this result is so important that we formulate in a proposition.

Proposition 1. *If energy is not used in production and nominal wages and goods prices cannot adjust, a flexible energy price and a monetary policy rule that sees through energy-price changes are sufficient to implement the efficient green transition. Hence, the increase in CPI that is generated by the increase in the energy price is exactly the increase that is needed to make sure that real wage and the real price of goods satisfy (34) and (35).*

Proof. See the Appendix. □

The implication of Proposition 1 is that, even in the extreme case where $P_{g,t}$ and W_t cannot adjust at all, a flexible energy price is all that is needed to implement the efficient transition when energy is not used in production. This is, however, no longer true in the more realistic scenario when energy is used in production as is stated in Proposition 2.

Proposition 2. *If energy is used in production and nominal wages and goods prices cannot adjust, a flexible energy price and a monetary policy rule that sees through energy-price changes are no longer sufficient to implement the efficient green transition. The*

¹⁶The r.h.s. of (35) is increasing in τ_t , but because the elasticity of substitution between fossil and green energy (ε) is larger than one, the increase of the marginal cost is bounded from above and finite even when $\tau \rightarrow 1$.

¹⁷The increase of the r.h.s. from the tax rate could, in principle, be offset by technological progress, which we abstract from here. Note, however, that technical change would need to be directed to increase the productivity of *fossil energy* and not *green*. Technological progress is potentially important, but we expect these processes to be relatively slow moving relative to price adjustments even if prices and wages are sticky. The implications of directed technical change is considered in Section 4.2.4

¹⁸This effect is captured in (34) by the product of γ and a square bracket.

resulting increase in CPI from the increase in the energy price results in a real relative price of goods that is too low and a real wage that is too high.

Proof. See the Appendix. □

In the full model with nominal frictions, $P_{g,t}$ and W_t are not fixed but they can adjust—if only in a staggered way. The implication of Proposition 2 is then that, when energy is used in production, at least one of these prices will indeed have to adjust when the carbon tax increases the price of energy. If nominal frictions are present, then the necessary nominal price changes will give rise to some costs. To evaluate exactly how large these costs will be requires the full model, but already here it is possible to infer some aspects of the costs.

To see this, consider an economy where nominal wages and goods prices cannot adjust at all as in Proposition 2. Assume further that the economy initially is in the steady state where the carbon tax is τ_0 . If the carbon tax then is increased to $\tau_t > \tau_0$, the resulting real wage in period t will be given by¹⁹

$$w_t = \left[(1 - \nu) \left(1 - \gamma + \gamma \left[(1 - \lambda) \left(\frac{1}{1 - \tau_0} \right)^{1 - \varepsilon} + \lambda \right]^{\frac{1 - \rho}{1 - \varepsilon}} \right)^{\frac{1 - \eta}{1 - \rho}} + \nu \left[(1 - \lambda) \left(\frac{1}{1 - \tau_t} \right)^{1 - \varepsilon} + \lambda \right]^{\frac{1 - \eta}{1 - \varepsilon}} \right]^{\frac{1}{\eta - 1}} .$$

The difference relative to (34) is that the tax rate τ_0 appears inside the second square bracket instead of the tax rate τ_t that would be efficient. In other words, the difference between the realized and the efficient real wage depends crucially on the parameter γ , which is the weight on energy in production, and the carbon tax. The weight on energy in production is by any estimate small: typically between 0.03 and 0.05. Such small numbers immediately reveal that the deviation of the real wage from the efficient real wage (given by (34)) is predicted to be small even in the presence of nominal frictions. The only way that the deviation could be non-trivial would be if the change in the carbon tax is large. However, if the carbon tax is phased in as is the case with the Fit for 55 package, deviations from efficiency will, at each point in time, be small. It is straightforward to verify that the same logic holds true for the price of goods.

Let us now consider some of the the inflationary consequences that could result from the green transition.

¹⁹See the proof of Proposition 2 in the Appendix.

4.1.2 Inflationary consequences

In the previous section, it was concluded that the real wage will fall in the transition. With falling income, also consumption will fall. Closed-form expressions for consumption and labor supply are provided in Appendix A.6. The reduction in consumption translates into a fall in the real interest rate, which from the Euler equation can be written as

$$R_{t+1} = \frac{1}{\beta} \mathbb{E}_t \left[\frac{U'(C_t)}{U'(C_{t+1})} \right].$$

With falling consumption, the marginal utility of consumption is increasing and, consequently, the real interest rate is falling. Once the tax rate reaches its new steady-state level, however, the real interest rate also reverts back to its steady state level that equals the inverse of the discount factor.

The inflationary consequence of these necessary adjustments will depend critically on how monetary policy is conducted and the extent to which the economy features sticky prices and wages. For now, we stay in the setting without nominal frictions and a central bank that follows a Taylor-type rule, and note that the nominal rate is linked to the real rate through a Fisher equation. In our perfect-foresight economy, the equation reads ²⁰

$$R_t = \frac{I_t}{\Pi_{t+1}}. \quad (37)$$

Combining (37) with the Taylor rule in (15) that targets CPI we get, after some manipulation, that current inflation is fully determined by (the expectations of) current and future real interest rates

$$\Pi_t = \prod_{j=0}^{\infty} (\beta R_{t+j})^{\frac{1}{\alpha_\pi}}. \quad (38)$$

Since all the entries in the product in (38) are below or equal to the pre-transitional values, it follows that the transition generates downward pressures on inflation. Exactly how much downward pressure that the green transition puts on inflation depends on how aggressive the Taylor rule is: the higher the weight on stabilizing inflation around the inflation target (α_π) is, the less downward pressure on inflation there will be. It is, in fact, possible to drive the inflationary response arbitrarily close to the target with a high enough weight. In the flex-price setting this has no cost, since nominal variables have no

²⁰Equation (37) can be derived by combining the real Euler equation with its nominal counterpart, (19), to get $R_t = E_t \frac{U'(C_{t+1})/\Pi_{t+1}}{U'(C_{t+1})} I_t$, and then noting that with perfect foresight the expectation term can be dropped.

real effects.²¹

Alternatively, the central bank could simply “see through” the energy price increase and instead focus exclusively on core inflation. In this case, monetary policy could be designed so that core inflation would not deviate at all from its target, which would imply a larger nominal price adjustment of energy in order to keep the relative price at the desired level. This would also imply a higher level of CPI inflation. Under flexible prices, however, the real effects would be unchanged.

With this intuition, let us now turn to the full transition while continuing to stay in a setting without nominal frictions and where the central bank follows the Taylor rule described by (15). The results are presented in Figure 1.

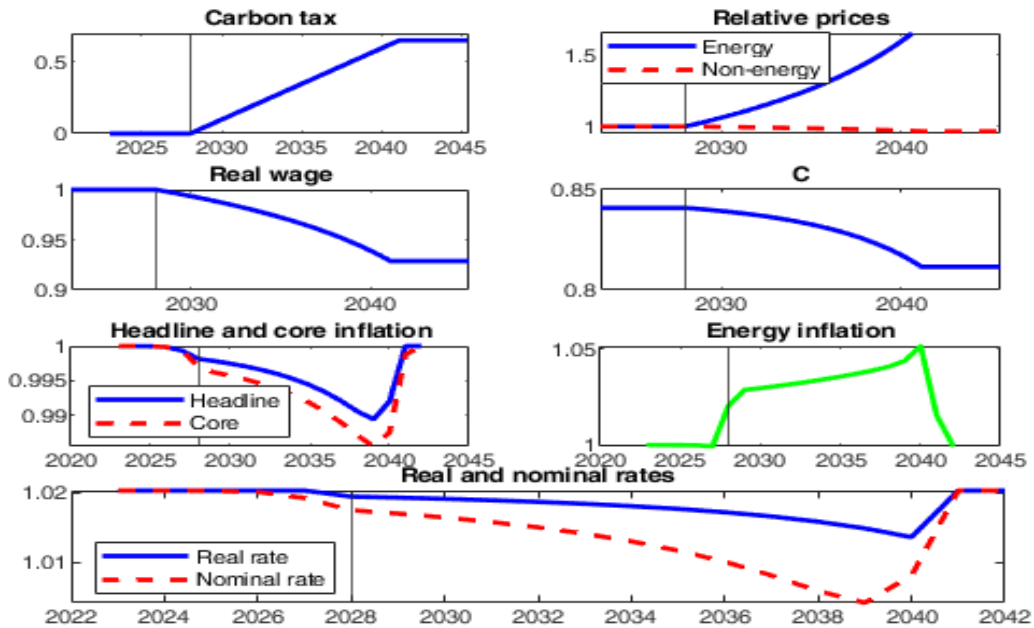


Figure 1: The effects of a green transition in an economy without nominal rigidities. The transition is announced 5 years in advance, and the vertical line denotes the first increase in the tax rate.

The top left graph plots the carbon tax that is implemented in the transition. All sub-plots are in line with the intuition provided above. The third row of graphs provide the inflationary consequences for headline (CPI), core (which in our setting is defined as headline inflation excluding energy inflation), and energy inflation.

We are now ready to sum up the results from this section: the green transition in a relatively small region requires the relative price of energy to increase and the relative price of goods, the marginal cost of producing goods, the real wage, output, and con-

²¹A similar result was independently derived in somewhat different model in Del Negro et al. (2023).

sumption to fall in that region. If energy is not used in production, a flexible energy price is sufficient to implement the efficient green transition. The reason is that $p_{g,t}$ ($P_{g,t}$) and w_t (W_t) always move one for one, which implies that the nominal price of energy can adjust to match all necessary relative prices. If energy is used in production then either $P_{g,t}$ and/or W_t also need to adjust to implement that efficient transition. However, if the carbon tax is phased in, the required nominal adjustments will be small.

4.2 Nominal Rigidities

We now turn to the implications with nominal rigidities. In contrast to the case without nominal rigidities, monetary policy can now have real effects on the economy. Hence, the exact specification of the central bank's objective becomes important.

We consider three cases: sticky prices and flexible wages, flexible prices and sticky wages, and sticky prices and wages. In all cases, we maintain the realistic assumption that energy prices are fully flexible.

4.2.1 Sticky goods prices and flexible wages

We now know from the previous section that the efficient green transition requires the relative price of goods and the real wage to decrease when the relative price of energy increases as a result of the carbon tax. If both the wage and the price of energy are fully flexible, the efficient green transition can still be implemented even though goods prices are sticky—but only if monetary policy is correctly designed.

Note, in particular, that a monetary policy rule that targets CPI inflation could never be efficient here. In this case, the nominal price of (non-energy) goods would have to fall when the energy price increases in order to make sure that the CPI inflation rate does not deviate from its target. If nominal goods prices are sticky so that not all these prices can change at once, then changing goods prices would result in a costly price dispersion that gives rise to an inefficient allocation of goods.

A well-known result in the monetary-policy literature then states that if price rigidities differ across sectors, it is optimal for the central bank to put higher weights on sectors with relatively more rigid prices.²² In particular, if a price is fully flexible—as we assume for the price of energy—its weight in the optimal inflation index should be zero. Because all goods prices in the model economy has the same degree of rigidity, the optimal price-index is thus CPI excluding energy.

With respect to the green transition, this implies that the central bank should make sure that core inflation remains on target. This policy eliminates the realization of costly

²²See Aoki (2001).

and inefficient price dispersions for goods. The nominal wage and the price of energy can then simply adjust so that the efficient green transition can be replicated. To see this, consider the following Taylor rule that targets core inflation

$$I_t = \frac{1}{\beta} \Pi_g^* \left(\frac{p_{g,t} \Pi_t}{p_{g,t-1} \Pi_g^*} \right)^{\alpha_\pi}. \quad (39)$$

The implications of this monetary-policy rule with a high value of α_π is illustrated in Figure 2. The top graphs verify the logic that just was provided: in this economy

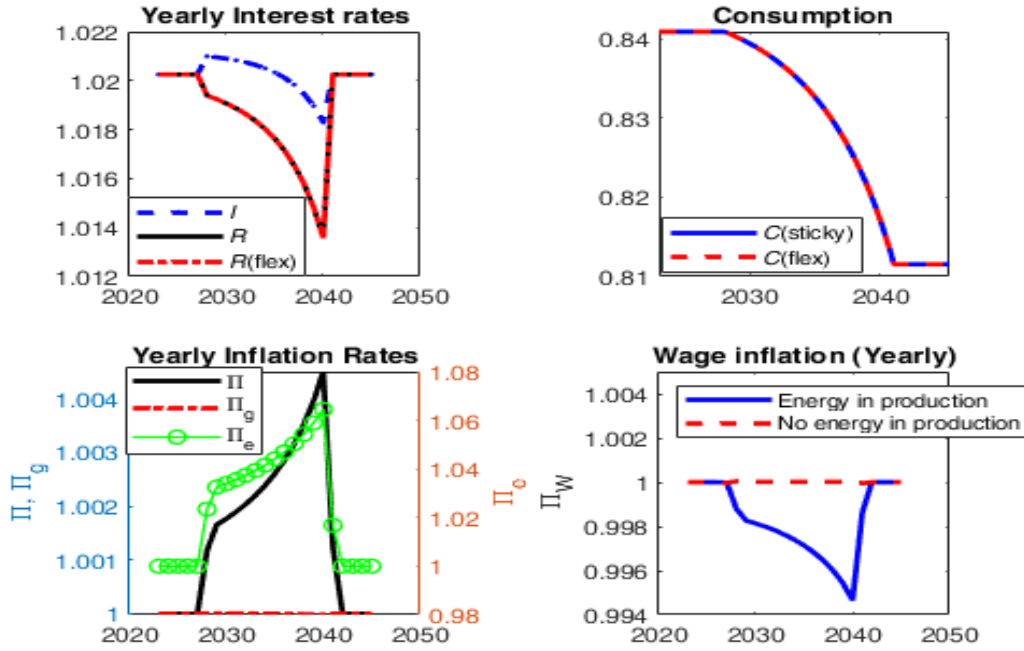


Figure 2: An economy with sticky prices, flexible wages, and a monetary-policy rule that targets core inflation. α_π is set to 100 to make sure that core inflation remains zero throughout the transition. Note that in the bottom left graph, Π_t and $\Pi_{g,t}$ uses the left y -axis, whereas $\Pi_{e,t}$ uses the right one. All plots except the bottom right one features energy in production.

a monetary-policy rule that ensures that core-inflation remains zero can replicate the efficient green transition. This is here illustrated by the fact that the real interest rate and consumption are both perfectly matched.²³

The bottom left graph then shows that the resulting increase in the energy price now should translate into an increase in CPI inflation. This effect is quantitatively small (the maximum yearly inflation rate stays below 0.5% for any year in the transition), but

²³Also labor supply is perfectly matched. The deviations from the flex-price solution can be made arbitrarily small as higher values of α_π are chosen.

more importantly: this inflation is not costly since we are replicating the efficient green transition.

Consistent with the intuition provided in Section 4.1.1, the bottom right graph shows that if energy is not an input into production, then wage inflation will also be exactly zero. Hence, even though the nominal wage can adjust, no adjustments will take place. The graph also shows that if energy is used, then the real wage will fall more than $p_{g,t}$ when the price of energy increases. The nominal wage then also need to fall.

To sum up the results from this section: if goods prices are sticky but nominal wages flexible, then it is optimal for the central bank to see through the rising energy prices during the green transition. By doing so the efficient green transition can be implemented. The result is a mild increase in CPI inflation, but this inflation is not costly.

4.2.2 Flexible goods prices and sticky wages

Consider now, instead, a setting where all prices are fully flexible, except the nominal wage that is subject to frictions. The logic provided in the previous section then still applies: with two flexible nominal prices it is possible to replicate the efficient green transition, but only if monetary policy is correctly designed.

Also in line with the findings in the previous section, a monetary policy rule that targets CPI inflation can never be efficient here. If the central bank aims at keeping CPI inflation on target, the nominal wages will have to adjust when the carbon tax increases. Because nominal wages are sticky and only can adjust in a staggered way, the result would be a wage distribution that gives rise to an inefficient allocation.

As first shown by Erceg et. al. (2000), it is, in this case optimal for the central bank to fully stabilize wage inflation, i.e., the growth-rate of nominal wages. With wage inflation given by $\Pi_W \equiv W_t/W_{t-1} = (w_t/w_{t-1}) \Pi_t$, it follows that the Taylor rule now should read

$$I_t = \frac{1}{\beta} \Pi_W^* \left(\frac{w_t}{w_{t-1}} \frac{\Pi_t}{\Pi_W^*} \right)^{\alpha_W}. \quad (40)$$

The results are not plotted but very similar to with sticky prices and flexible wages. The Taylor rule described in (40), calibrated with a high enough value of α_W that ensures that wage-inflation remains zero while goods and energy prices can adjust so that the efficient green transition is implemented.²⁴

We, again, know from Section 4.1.1 that, if energy is not used as an input in production, then the goods-price inflation will also be zero. However, if energy is used in production, then goods-prices will have to increase to make sure that the real wage falls

²⁴The deviations from the flex-price solution can be made arbitrarily small as higher values of α_W are chosen.

by more than the real goods price. This will result in a slightly higher CPI inflation relative to with sticky prices and flexible wages.²⁵ Again, however, this inflation is not costly since the the efficient green transition is implemented.

4.2.3 Sticky goods prices and sticky wages

Finally, we now consider the case where both prices of non-energy goods and wages are rigid. We already know from Section 4.1 that if energy is not used in production, a flexible energy price and a central bank that sees through the increase in the energy price is enough to implement the efficient green transition. However, in the more realistic case when energy is used in production, this will not be enough. The price of energy can then either adjust so that the relative price between energy and non-energy is correct, or so that the relative price between energy and wages is correct – but not so that both these relative prices are correct at each point in time. Hence, either the nominal wage, the nominal price of goods or a combination of both will have to adjust.

For that reason, an optimal monetary policy is going to have to accept some deviations of inflation for non-energy goods and/or wages. The central bank may face a trade-off between wage and price rigidities where the more rigid wages are and the more costly wage dispersion is perceived to be, the larger the share of the adjustment will go through goods price inflation and vice versa. To allow for such trade-off, we here consider the following Taylor rule that incorporates both core and wage inflation:

$$I_t = \frac{1}{\beta} \left[\Pi_g^* \left(\frac{\Pi_{g,t}}{\Pi_g^*} \right)^{\alpha_\pi} \Pi_W^* \left(\frac{w_t}{w_{t-1}} \frac{\Pi_t}{\Pi_W^*} \right)^{\alpha_w} \right]. \quad (41)$$

The results from this rule are presented in Figures 3-4. The former figure verifies the claim in Proposition 1 that, without energy in production, it is possible to implement the efficient green transition even though only the price of energy is flexible and the central bank sees through the increase in the energy price. For instance, the central bank can employ a Taylor rule that makes sure that either goods or wage inflation does not deviate from target. As can be seen, the efficient green transition is perfectly matched and the resulting increase in CPI is modest and peaks just above 0.5 percent. Figure 4 plots the transition when energy is used in production. As was conjectured above, the differences to when energy is not used in production are quantitatively small. With the use of a Taylor rule that gives a high weight on keeping wage inflation on target while allowing the price of goods to adjust, it is possible to closely replicate the efficient green transition. Consumption, labor supply and the real rate are all matched almost perfectly. As presented in Table 2, the median deviation of consumption from the efficient level

²⁵The maximum yearly inflation rate stays below one percent.

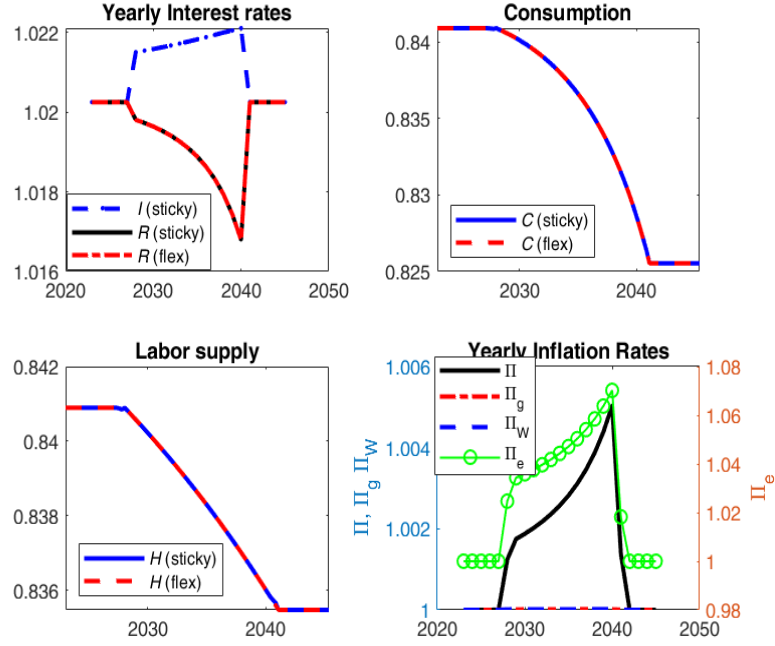


Figure 3: An economy without energy in production but with sticky prices and wages. α_π and α_W are respectively set to 1.5 and 100. Note that in the bottom right graph, Π_t and $\Pi_{g,t}$ uses the left y -axis, whereas $\Pi_{e,t}$ uses the right one.

is as small as 0.002 percent and the maximum deviation is 0.02, which quantitatively is negligible.

Note that the price of goods increase in the transition so that w falls by more than p_g . As with sticky wages and flexible goods prices, this results in a slightly higher CPI level, but the CPI peaks well below one percent.

It is, however, important that monetary policy is correctly designed in the transition. As shown in Table 2, if the central bank instead were to follow a Taylor rule that targets CPI, there would be sizeable deviations from efficiency.

Table 2: Deviations in C from the efficient green transition with nominal frictions

TAYLOR RULE	CPI	CORE	CORE AND WAGE
MAX DEVIATION (in %)	1.19	0.40	0.02
MEDIAN DEVIATION (in %)	0.14	0.11	0.002
α_π^*	1.25	1.5	1.5
α_W^*	-	-	17.5

Parameters α_π^* and α_W^* , respectively, denote the values that minimizes the maximum deviation for the considered Taylor rules.

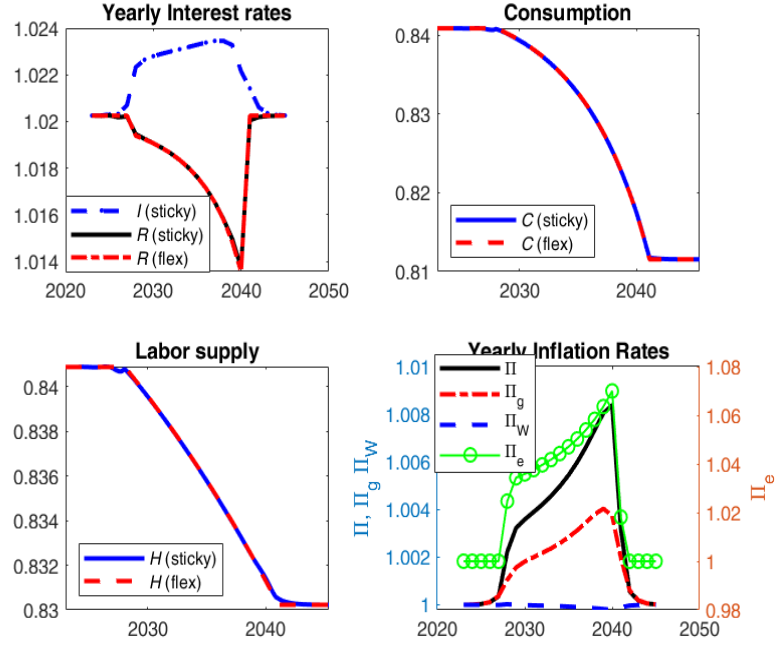


Figure 4: An economy with energy in production and sticky prices and wages. α_π and α_W are respectively set to 1.5 and 17.5. Note that in the bottom right graph, Π_t and $\Pi_{g,t}$ uses the left y -axis, whereas $\Pi_{e,t}$ uses the right one.

4.2.4 Faster green technological progress

We now consider the implications of faster green exogenous technological progress. It is very likely that more R&D resources will be directed at improving the green energy efficiency when policy seriously reduces the possibilities to use fossil fuel.²⁶

To evaluate the implications of green directed technical change, we feed in an exogenous increasing path for A_2 , which is the TFP component in green energy production. The process is calibrated so that A_2 grows at two percent per year for 18 years after which A_2 does not grow any more. Because the productivity in fossil-energy production is stagnant, the ratio of productivity in green energy production relative to that in fossil-energy production then also grows at two percent per year. The results are presented in Figure 5.

As can be seen, improvements in green-energy efficiency induces the price of energy to increase less than without technological progress. The difference is large: without technological progress the energy price is about 17% higher than with. The opposite is true for goods prices, but the difference is much smaller. The goods price without technological progress is only 1% lower than with. As a result, technical change that improves

²⁶See for instance, Acemoglu et al. (2012) and Hassler, Krusell, and Olovsson (2021).

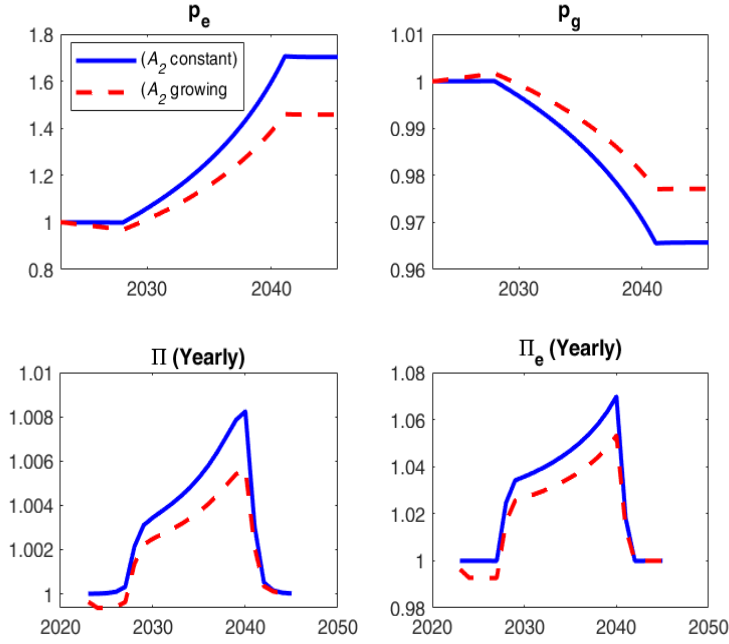


Figure 5: Price changes in the green transition when technical change is directed at improving the green energy efficiency. Goods prices and wages are both sticky and the calibration is the same as in Section 4.2.3.

the green-energy efficiency further reduces the already limited inflationary pressures from the green transition. The CPI now peaks at about 0.5% above target, i.e., the peak in inflation is reduced to 60% of the peak without green growth.

4.2.5 A transition that starts immediately

Figure 6 shows the implications of a transition where the tax is increased immediately when the transition is announced so that agents have no time to adjust in advance. Specifically, the figure plots results for when monetary policy follows a Taylor rule that incorporates both core and wage inflation (41) and the calibration is the same as in Section 4.2.3.

Clearly, the adjustment period to the transition is important. In contrast to the case with an adjustment period, the CPI now spikes around five percent above target in the beginning of the transition. The median deviation in consumption from the efficient solution is relatively small, but the maximum deviation is non-trivial at about 0.5 percent.

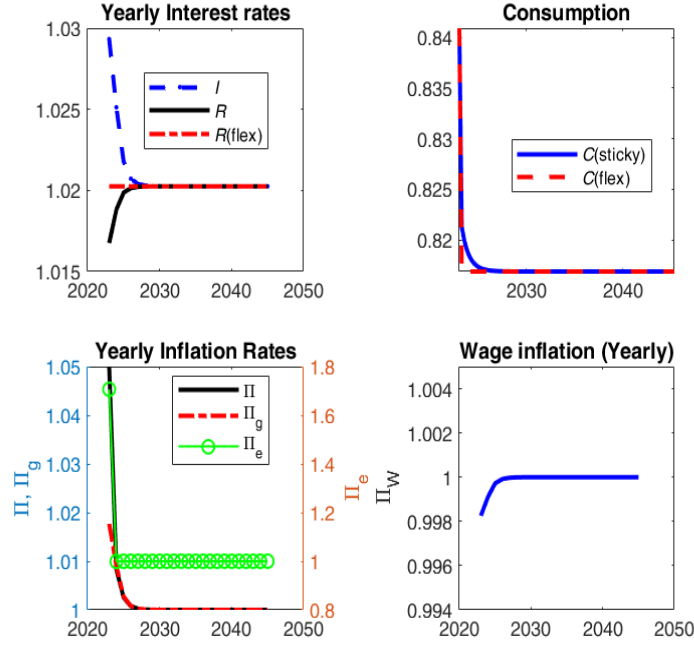


Figure 6: An immediate transition in an economy with energy in production and sticky prices and wages.

5 Discussion

Below we now discuss the implications of some potentially important features that have been omitted from the model.

5.1 Taylor rules versus optimal policy

As already mentioned in the Introduction, we employ simple Taylor rules for ease of exposition. Notice, however, that we in many cases are able to exactly replicate the flexible-price allocation, which in our case is constrained efficient. Specifically, the climate externality prevents a fully efficient solution since monetary policy is unable to influence the global coordination failure (which prevents a fully efficient solution). An alternative would thus be to instead consider a loss-function based policy, which would then result in the same equilibrium. However, by using optimal policy it would be possible to design more sophisticated rules that also allow for additional shocks.

5.2 The central-bank mandate

Even though it is uncontroversial in the theoretical literature that a central bank should target nominal wage growth when wages are sticky, this is not a mandate that tends to be

observed in the real world. There is, however, a small but growing literature that points out that normative (“unconventional”) monetary policy objectives such as wage-growth are strongly correlated with real activity measures such as the output gap.²⁷ Hence, a wage-growth target may loosely be thought of as having a high weight on the output-gap in a dual-mandate type of monetary policy strategy.

5.3 Green and brown capital

We have intentionally omitted green investment from the analysis. Specifically, we assume that the stock of capital in the brown and green energy-producing sectors are fixed and that the two energy inputs are combined with a CES aggregator to producing energy services. The assumption that capital stocks are fixed is clearly reasonable in the short-run, but may be less appropriate for the longer run.

When brown energy is taxed, its relative price increases and the demand for green energy goes up. If the capital stock is unable to adjust, the result is a reduction in the marginal productivity of green energy as well as an overall decrease in the productivity of the energy producing sector. These price/cost effects will be larger relative to a situation where additional investment could have increased the green capital stock. Hence, our assumption of a fixed capital allocation for the two energy sectors tends to overstate the price effects. We therefore only expect to see even lower inflationary pressures if investments could adjust to reduce brown and increase green capital. In fact, because capital enters the model in a very similar way as technological progress, we conjecture that the results would be similar to those from directed technical change (Section 4.2.4), i.e., inflationary effects would be even smaller. Since we find small inflation effects to begin with, we view our abstraction green capital as a conservative assumption.

One potential counter example could be if there would be a strong initial investment-boom in response to the announcement of the planned increase in taxes. For instance, if the government would step in and provide large subsidies to the production of green energy capital. In this case, a rapid increase in those subsidies would act as a conventional spending shock, which through standard channels could lead to some initial inflation. We leave such extensions of the model for future research.

5.4 Carbon taxes versus green subsidies

Some countries such as the United States seem to pursue an alternative approach to carbon taxation (or a cap-and-trade system) by instead considering subsidising investment in green energy production. While this approach will contribute to lowering the cost of

²⁷See e.g. Debortili et. al. (2018).

green energy, its success in reducing overall emissions is far less certain. The reason is that the key to reducing overall emissions is to make brown energy unprofitable—which is not a statement relative to green energy—but in relation to the marginal cost of producing brown energy. Unless the green subsidies drives down the price of brown energy below its marginal cost, the effect of the subsidies could just be to increase the demand for energy services in general and possibly a reduction of profits in the brown sector. This policy does, however, not necessarily lead to a reduction in carbon-dioxide emissions and could, in fact, even increase emissions. An explicit analysis and an in-depth discussion of carbon taxes versus green subsidies is provided in Hassler, Krusell, and Olovsson (2023).

Nevertheless, if green subsidies induces lower energy prices, this may affect inflationary pressures. Analogous to in our analysis above, the green policy will then require some real prices to adjust. As long as monetary policy can make the necessary adjustments, inflation will not have to deviate (much) from the target. As in a standard model, however, a large fiscal expansion may still have inflationary effects through the usual channels associated with fiscal policy. It would be straightforward to incorporate green subsidies into the model but, for now, we leave this for future research.

5.5 Climate change

Let us now finally comment on how climate change could be incorporated into the model. Following Golosov et al. (2014), the stock of carbon dioxide in the atmosphere, S , follows the process

$$S_{t+1} = (1 - \delta_s) \sum_{j=0}^{\infty} E_{1,t-j},$$

where δ_s is the depreciation of carbon dioxide from the atmosphere. Moreover, the damages could then be modelled as reducing the total factor productivity in each production sector, i.e.,

$$A_{g,t} = \bar{A}_{g,t} \exp(-\varrho S_t), \quad (42)$$

$$A_{1,t} = \bar{A}_{1,t} \exp(-\varrho S_t), \quad (43)$$

$$A_{2,t} = \bar{A}_{2,t} \exp(-\varrho S_t), \quad (44)$$

where \bar{A} , \bar{A}_1 , and \bar{A}_2 denote the total factor productivities that would prevail without any climate damages, and ϱ determines the size of the externality.

We do not solve this specification of the model, but just note that with climate externalities some of the results from the above sections could change slightly. For instance, if the whole world internalizes the externality, the real wage, consumption, and GDP could instead be increasing during the transition relative to a scenario without taxation. In

the absence of nominal frictions, for instance, there would instead be a slight increase in inflation if the central bank of the world targets CPI. Important to note, however, is that the main conclusions do not change. It would, still be possible for the central bank to achieve similar results as without climate change. Specifically, some specific numbers can change but not the overall principles derived.

6 Conclusions

We have analysed the real and nominal implications of a green transition to a state with sustainable energy production, and how should monetary policy react during such transition. Using a New-Keynesian model with an energy and a goods sector, we show that a green transition requires the relative price of energy to increase and the relative price of goods, the marginal cost of production, and the real wage to fall. We prove, in our setting, that if energy is not used in production and nominal wages and goods prices are rigid, a flexible energy price and a monetary policy rule that sees through energy-price changes can replicate the flex-price economy. In the more realistic case where energy is used in production there will be deviations from efficiency. However, because energy's share of income is small, these deviations are only marginal unless monetary policy is ill suited and/or if the increase in the carbon tax is aggressive and immediate. During the green transition, it is optimal for monetary policy to see through the increasing energy prices and focus on core inflation. The result is a modest increase in CPI.

7 References

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A Appendix

A.1 The consumer problem

The intra-temporal problem for the consumer is

$$\min_{C_{g,t}, C_{e,t}} P_{g,t}C_{g,t} + P_{e,t}C_{e,t} - P_t \left[C_t - \left((1 - \omega)^{\frac{1}{\eta}} C_{g,t}^{\frac{\eta-1}{\eta}} + \omega^{\frac{1}{\eta}} C_{e,t}^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} \right].$$

The first-order condition w.r.t. $C_{g,t}$ and $C_{e,t}$ can, respectively, be written as (17) and (18). Inserting (17) and (18) into (2) delivers (20).

The intertemporal problem for the consumer is to maximize (1) subject to (3). The resulting first-order conditions gives (19).

A.2 Firms

A.2.1 The goods sector

The cost minimization problem for the firm that assembles the manufactured final good is

$$\min_{Y_{g,t}(i)} P_{g,t}(i) Y_{g,t}(i) - P_{g,t} \left[Y_{g,t} - \left(\int_0^1 \left(Y_{g,t}(i) \right)^{\frac{\xi-1}{\xi}} di \right)^{\frac{\xi}{\xi-1}} \right].$$

The first order condition w.r.t. $Y_{g,t}(i)$ delivers (6).

Turning to the problem for firm i , we have

$$\min_{H_{g,t}, e_{g,t}} W_t H_{g,t}(i) + P_{e,t} e_{g,t}(i) - MC_{g,t} \left(\left[(1-\gamma)^{\frac{1}{\rho}} (A_g H_{g,t}(i))^{\frac{\rho-1}{\rho}} + \gamma^{\frac{1}{\rho}} (e_{g,t}(i))^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}} - Y_{g,t}(i) \right),$$

where, by construction, the Lagrange multiplier, $MC_{g,t}$, defines the exact marginal cost for inputs.

Foc $H_{g,t}(i)$:

$$H_{g,t}(i) = (1-\gamma) \frac{1}{A_{g,t}} \left(\frac{W_t}{A_{g,t} MC_{g,t}} \right)^{-\rho} Y_{g,t}(i); \quad (\text{A-1})$$

foc $e_{g,t}(i)$:

$$e_{g,t}(i) = \gamma \left(\frac{P_{e,t}}{MC_{g,t}} \right)^{-\rho} Y_{g,t}(i). \quad (\text{A-2})$$

Back into the constraint delivers

$$MC_{g,t} = \left[(1-\gamma) \left(\frac{W_t}{A_{g,t}} \right)^{1-\rho} + \gamma P_{e,t}^{1-\rho} \right]^{\frac{1}{1-\rho}},$$

with the real marginal cost is given by

$$mc_{g,t} \equiv \frac{MC_{g,t}}{P_t} = \left[(1-\gamma) \left(\frac{w_t}{A_t} \right)^{1-\rho} + \gamma p_{e,t}^{1-\rho} \right]^{\frac{1}{1-\rho}}.$$

Notice that the real marginal cost is independent of the level of production, since the relative price of energy and the wage rate are exogenous from the viewpoint of the small firm.

The first-order condition to (8) can be written as

$$\mathbb{E}_t \sum_{j=0}^{\infty} \theta^j Q_{t,t+j} \left(Y_{g,t+j}(i) + (P_{g,t}^* - MC_{g,t+j}(Y_{g,t+j}(i))) \frac{\partial Y_{g,t+j}(i)}{\partial P_{g,t}^*} \right) = 0,$$

where $MC_{g,t+j}$ is the marginal cost of production in period $t+j$. Noting that $\frac{\partial Y_{g,t+j}(i)}{\partial P_{g,t}^*} = -\frac{\xi}{P_{g,t}^*} Y_{g,t+j}(i)$, we get

$$\mathbb{E}_t \sum_{j=0}^{\infty} \theta^j Q_{t,t+j} Y_{g,t+j}(i) \left[P_{g,t}^* - \frac{\xi}{\xi-1} MC_{g,t+j} \right] = 0. \quad (\text{A-3})$$

Inserting the stochastic discount factor $Q_{t,t+j} = \beta^j \frac{U'(C_{t+j})}{U'(C_t)} \frac{P_t}{P_{t+j}}$ and (9), we arrive at

$$\begin{aligned} & \frac{P_{g,t}^*}{P_{g,t}} E_t \sum_{j=0}^{\infty} (\beta\theta)^j U'(C_{t+j}) \frac{P_t}{P_{t+j}} \left(\frac{P_{g,t+j}}{P_{g,t}} \right)^\xi Y_{g,t+j} \\ &= \frac{\xi}{\xi-1} \frac{P_t}{P_{g,t}} E_t \sum_{j=0}^{\infty} (\beta\theta)^j U'(C_{t+j}) \frac{W_{t+j}}{P_{t+j}} \frac{1}{A_{g,t+j}} \left(\frac{P_{g,t+j}}{P_{g,t}} \right)^\xi Y_{g,t+j}. \end{aligned} \quad (\text{A-4})$$

Now define

$$N_{1,t} = E_t \sum_{j=0}^{\infty} (\beta\theta)^j U'(C_{t+j}) \frac{P_t}{P_{t+j}} \left(\frac{P_{g,t+j}}{P_{g,t}} \right)^\xi Y_{g,t+j},$$

which implies that

$$N_{1,t+1} = E_{t+1} \sum_{j=0}^{\infty} (\beta\theta)^j U'(C_{t+j+1}) \frac{P_{t+1}}{P_{t+j+1}} \left(\frac{P_{g,t+j+1}}{P_{g,t+1}} \right)^\xi Y_{g,t+j+1}.$$

Then take expectations, using the law of iterated expectations and subtracting this from $K_{1,t}$ gives that

$$\begin{aligned} N_{1,t} &= U'(C_t) Y_{g,t} + E_t \frac{P_t}{P_{t+1}} \left(\frac{P_{g,t+1}}{P_{g,t}} \right)^\xi \beta\theta N_{1,t+1}, \text{ or} \\ N_{1,t} &= U'(C_t) Y_{g,t} + \beta\theta E_t \Pi_{t+1}^{-1} \Pi_{g,t+1}^\xi N_{1,t+1}. \end{aligned}$$

Similarly,

$$\begin{aligned} N_{2,t} &= E_t E_t \sum_{j=0}^{\infty} (\beta\theta)^j U'(C_{t+j}) \frac{MC_{t+j}}{P_{t+j}} \left(\frac{P_{g,t+j}}{P_{g,t}} \right)^\xi Y_{g,t+j}, \text{ or} \\ N_{2,t} &= U'(C_t) mc_t Y_{g,t} + \beta\theta E_t \Pi_{g,t+1}^\xi N_{2,t+1}. \end{aligned}$$

We can now express

$$\frac{P_{g,t}^*}{P_{g,t}} N_{1,t} = \frac{\xi}{(\xi-1)(1+\tau_g)} \frac{P_t}{P_{g,t}} N_{2,t}, \quad (\text{A-5})$$

where τ_g is a tax rate used to offset the distortion generated by market power. Specifically, we set τ_g to

$$\tau_g = \frac{\xi}{\xi-1} - 1,$$

which ensures that the equilibrium is identical to the flex-price equilibrium if the nominal frictions are set to zero.

Next, since $P_{g,t}^{1-\xi} = (1-\theta)(P_{g,t}^*)^{1-\xi} + \theta P_{g,t-1}^{1-\xi}$, we see that

$$\frac{P_{g,t}^*}{P_{g,t}} = \left[\frac{1 - \theta \Pi_{g,t}^{\xi-1}}{1 - \theta} \right]^{\frac{1}{1-\xi}}. \quad (\text{A-6})$$

Inserting (A-6) into (A-5) delivers (23).

To solve the model, we make use of the following manipulation

$$\begin{aligned} D_{g,t} &\equiv \int_0^1 \left(\frac{P_{g,t}(i)}{P_{g,t}} \right)^{-\xi} di, \\ D_{g,t} &= \sum_{j=0}^{\infty} (1-\theta) \theta^j \left(\frac{P_{g,t-j}^*}{P_{g,t}} \right)^{-\xi}, \\ D_{g,t-1} &= \sum_{j=0}^{\infty} (1-\theta) \theta^j \left(\frac{P_{g,t-1-j}^*}{P_{g,t-1}} \right)^{-\xi} \\ \theta \left(\frac{P_{g,t-1}}{P_{g,t}} \right)^{-\xi} D_{g,t-1} &= \sum_{j=1}^{\infty} (1-\theta) \theta^j \left(\frac{P_{g,t-j}^*}{P_{g,t}} \right)^{-\xi}, \\ D_{g,t} - \theta \left(\frac{P_{g,t-1}}{P_{g,t}} \right)^{-\xi} D_{g,t-1} &= (1-\theta) \left(\frac{P_{g,t}^*}{P_{g,t}} \right)^{-\xi}, \\ D_{g,t} &= (1-\theta) \left(\frac{P_{g,t}^*}{P_{g,t}} \right)^{-\xi} + \theta \Pi_{g,t}^{\xi} D_{g,t-1}, \end{aligned}$$

Finally, from (A-1), (A-2), and (25), we then have

$$\begin{aligned} H_{g,t}(i) &= (1-\gamma) \frac{1}{A_{g,t}} \left(\frac{w_t}{A_{g,t} m c_t} \right)^{-\rho} Y_{g,t}(i), \\ e_{g,t}(i) &= \gamma \left(\frac{p_{e,t}}{m c_t} \right)^{-\rho} Y_{g,t}(i). \end{aligned}$$

Hence

$$H_{g,t} = (1-\gamma) \frac{D_{g,t}}{A_{g,t}} \left(\frac{w_t}{A_{g,t} m c_t} \right)^{-\rho} Y_{g,t},$$

and

$$e_{g,t} = \gamma D_{g,t} \left(\frac{p_{e,t}}{m c_t} \right)^{-\rho} Y_{g,t}.$$

A.3 Unions

To solve (14), substitute the constraints into the utility function and take the first order conditions to arrive at

$$\mathbb{E}_t \sum_{k=0}^{\infty} (\beta\theta_w)^k \left(\frac{1}{C_{t+k}} \frac{1}{P_{t+k}} \left(H_{t+k|t}(j) + W_t^* \frac{\partial H_{t+k|t}(j)}{\partial W_t^*} \right) - H_{t+k|t}(j)^\varphi \frac{\partial H_{t+k|t}(j)}{\partial W_t^*} \right) = 0.$$

Using

$$\frac{\partial H_{t+k|t}(j)}{\partial W_t^*} = -\frac{\xi_w}{W_t^*} H_{t+k|t}(j),$$

we get

$$0 = \mathbb{E}_t \sum_{k=0}^{\infty} (\beta\theta_w)^k \left(W_t^* \frac{1}{C_{t+k}} \frac{1}{P_{t+k}} H_{t+k|t}(j) - \frac{\xi_w}{\xi_w - 1} H_{t+k|t}(j)^{1+\varphi} \right).$$

Substitute out $H_{t+k|t}$ and manipulate to get an expression in terms of real wages instead of nominal

$$\mathbb{E}_t \sum_{k=0}^{\infty} (\beta\theta_w)^k \left(\left(\frac{W_t^*}{C_{t+k} P_{t+k}} \right)^{1+\xi_w \varphi} \left(\frac{1}{P_{t+k} w_{t+k}} \right)^{-\xi_w} H_{t+k} - \frac{\xi_w}{\xi_w - 1} \left(\left(\frac{1}{P_{t+k} w_{t+k}} \right)^{-\xi_w} H_{t+k} \right)^{1+\varphi} \right) = 0.$$

Now, multiply with P_t inside the parentheses with P_t/P_t to get

$$\mathbb{E}_t \sum_{k=0}^{\infty} (\beta\theta_w)^k \left((w_t^*)^{1+\xi_w \varphi} 1 \frac{P_t^{(1+\xi_w \varphi) + \xi_w - \xi_w(1+\varphi)}}{C_{t+k} P_{t+k}} \left(\frac{\Pi_{t,t+k}^{-1}}{w_{t+k}} \right)^{-\xi_w} H_{t+k} - \frac{\xi_w}{\xi_w - 1} \left(\left(\frac{\Pi_{t,t+k}^{-1}}{w_{t+k}} \right)^{-\xi_w} H_{t+k} \right)^{1+\varphi} \right) = 0.$$

Because $(1 + \xi_w \varphi) + \xi_w - \xi_w(1 + \varphi) = 1$, we arrive at

$$\mathbb{E}_t \sum_{k=0}^{\infty} (\beta\theta_w)^k \left((w_t^*)^{1+\xi_w \varphi} \frac{1}{C_{t+k}} \Pi_{t,t+k}^{\xi_w - 1} w_{t+k}^{\xi_w} H_{t+k} - \frac{\xi_w}{\xi_w - 1} \left(\Pi_{t,t+k}^{\xi_w} w_{t+k}^{\xi_w} H_{t+k} \right)^{1+\varphi} \right) = 0,$$

$$(w_t^{*w})^{1+\xi_w \varphi} = \frac{\xi_w}{\xi_w - 1} \frac{M_{1,t}}{M_{2,t}}$$

with

$$M_{1,t} = \mathbb{E}_t \sum_{k=0}^{\infty} (\beta\theta_w)^k \left(\Pi_{t,t+k}^{\xi_w} w_{t+k}^{\xi_w} H_{t+k} \right)^{1+\varphi}$$

$$M_{2,t} = \mathbb{E}_t \sum_{k=0}^{\infty} (\beta\theta_w)^k \frac{1}{C_{t+k}} \Pi_{t,t+k}^{\xi_w-1} w_{t+k}^{\xi_w} H_{t+k}.$$

Recursify, take expectations at t , and subtract the result from $M_{1,t}$, to arrive at (A-16)-(A-17).

The aggregate wage index follows a similar logic as the price index:

$$W_t^{1-\xi_w} = (1 - \theta_w) (W_t^*)^{1-\xi_w} + \theta_w W_{t-1}^{1-\xi_w}.$$

Dividing by $P_t^{1-\xi_w}$ to get (27).

A.3.1 Energy production and supply

The first-order conditions to (10), and (11) are given by

$$W_t = (1 - \tau_t) P_{1,t} A_{1,t}, \quad (\text{A-7})$$

and

$$W_t = P_{2,t} A_{2,t}. \quad (\text{A-8})$$

$$P_{1,t} = P_{2,t} \frac{A_{2,t}}{A_{1,t}} \frac{1}{1 - \tau_t}. \quad (\text{A-9})$$

The cost-minimization problem for the energy service provider, (13), delivers the following equilibrium conditions

$$E_{1,t} = \left(\frac{P_{1,t}}{P_{e,t}} \right)^{-\varepsilon} (1 - \lambda) (C_{e,t} + e_{g,t}), \quad (\text{A-10})$$

$$E_{2,t} = \left(\frac{P_{2,t}}{P_{e,t}} \right)^{-\varepsilon} \lambda (C_{e,t} + e_{g,t}), \quad (\text{A-11})$$

and

$$P_{e,t} = h_t P_{2,t}, \quad (\text{A-12})$$

where

$$h_t \equiv \left[(1 - \lambda) \left(\frac{A_{2,t}}{A_{1,t}} \frac{1}{1 - \tau_t} \right)^{1-\varepsilon} + \lambda \right]^{\frac{1}{1-\varepsilon}}. \quad (\text{A-13})$$

Using (A-9) and (A-12) in (A-10) and (A-11), these latter expressions can be formulated as (21) and (22).

A.4 Help terms

$$N_{1,t} = U'(C_t) Y_{g,t} + \beta \theta E_t \Pi_{t+1}^{-1} \Pi_{g,t+1}^\xi N_{1,t+1}, \quad (\text{A-14})$$

$$N_{2,t} = U'(C_t) Y_{g,t} m c_t + \beta \theta E_t \Pi_{g,t+1}^\xi N_{2,t+1}. \quad (\text{A-15})$$

$$M_{1,t} = \left(w_t^{\xi w} H_t \right)^{1+\varphi} + \beta \theta_w E_t \Pi_{t+1}^{\xi w (1+\varphi)} M_{1,t+1}, \quad (\text{A-16})$$

$$M_{2,t} = \frac{1}{C_t} w_t^{\xi w} H_t + \beta \theta_w E_t \Pi_{t+1}^{\xi w - 1} M_{2,t+1}. \quad (\text{A-17})$$

$$D_{g,t} = (1 - \theta) \left[\frac{1 - \theta \Pi_{g,t}^{\xi-1}}{1 - \theta} \right]^{\frac{\xi}{\xi-1}} + \theta \Pi_{g,t}^\xi D_{g,t-1}. \quad (\text{A-18})$$

$$\Pi_{g,t} = \Pi_t \frac{p_{g,t}}{p_{g,t-1}} \quad (\text{A-19})$$

$$\Pi_{e,t} = \Pi_t \frac{p_{e,t}}{p_{e,t-1}}, \quad (\text{A-20})$$

where we used (A-6) to arrive at (A-18).

We also have

$$\widehat{T}_t \equiv \frac{T_t}{P_t} = \frac{A_{2,t}}{h_t} p_{e,t} H_{1,t} \frac{\tau_t}{1 - \tau_t} + \frac{I_{t-1}}{\Pi_t} b_{t-1} - b_t. \quad (\text{A-21})$$

A.5 Steady state

The steady-state versions of (15) and (19) are

$$I = \frac{1}{\beta} \Pi^* \left(\frac{\Pi}{\Pi^*} \right)^{\alpha \pi}$$

and

$$I = \Pi \frac{1}{\beta}.$$

Combining these two equations delivers

$$\Pi = \Pi^*; \quad (\text{A-22})$$

$$I = \frac{1}{\beta} \Pi. \quad (\text{A-23})$$

We then have

$$\Pi_g = \Pi_e = \Pi; \quad (\text{A-24})$$

$$D_g = \frac{1 - \theta}{1 - \theta \Pi_{g,t}^\xi} \left[\frac{1 - \theta \Pi_{g,t}^{\xi-1}}{1 - \theta} \right]^{\frac{\xi}{\xi-1}}; \quad (\text{A-25})$$

$$w = A_2 p_e \frac{1}{h}; \quad (\text{A-26})$$

$$h \equiv \left[(1 - \lambda) \left(\frac{A_2}{A_1} \frac{1}{1 - \tau} \right)^{1-\varepsilon} + \lambda \right]^{\frac{1}{1-\varepsilon}}; \quad (\text{A-27})$$

$$(1 - \nu) (p_g)^{1-\eta} + \nu (p_e)^{1-\eta} = 1; \quad (\text{A-28})$$

Using the steady-state expressions of (A-14) and (A-15) in (23), we get the steady-state version of the latter equation

$$p_g = \frac{\xi}{\xi - 1} \frac{Mc}{P} \left[\frac{1 - \theta \Pi_g^{\xi-1}}{1 - \theta} \right]^{\frac{1}{\xi-1}} \frac{1 - \beta \theta \Pi^{-1} \Pi_g^\xi}{1 - \beta \theta \Pi_g^\xi}; \quad (\text{A-29})$$

$$mc = \left[(1 - \gamma) \left(\frac{w}{A_g} \right)^{1-\rho} + \gamma p_e^{1-\rho} \right]^{\frac{1}{1-\rho}};$$

Inserting the steady-state versions of (A-16)-(A-17) into (26) and then into (27) and rearranging allows us to solve for H , we get

$$H = \left[\frac{1 - \beta \theta_w \Pi^{\xi_w(1+\varphi)}}{1 - \beta \theta_w \Pi^{\xi_w-1}} \frac{\xi_w - 1}{\xi_w} \frac{w}{C} \left(\frac{1 - \theta_w}{1 - \theta_w \Pi^{\xi_w-1}} \right)^{\frac{1+\xi_w \varphi}{\xi_w-1}} \right]^{\frac{1}{\varphi}}; \quad (\text{A-30})$$

$$M_1 = \frac{(w^{\xi_w} H)^{1+\varphi}}{1 - \beta \theta_w \Pi^{\xi_w(1+\varphi)}}; \quad (\text{A-31})$$

$$M_2 = \frac{w^{\xi_w} H}{C (1 - \beta \theta_w \Pi^{\xi_w-1})}; \quad (\text{A-32})$$

$$w_t^* = \left(\frac{\xi_w}{\xi_w - 1} \frac{M_{1,t}}{M_{2,t}} \right)^{\frac{1}{1+\xi_w \varphi}} \quad (\text{A-33})$$

$$\left(\nu p_e^{-\eta} + \gamma D_g \left(\frac{p_e}{mc} \right)^{-\rho} (1 - \nu) p_g^{-\eta} \right) h_2$$

$$(1 - \gamma) \frac{D_g}{A_g} \left(\frac{w}{A_g mc} \right)^{-\rho} (1 - \nu) p_g^{-\eta} + \left(\nu p_e^{-\eta} + \gamma D_g \left(\frac{p_e}{mc} \right)^{-\rho} (1 - \nu) p_g^{-\eta} \right) h_2 = \frac{H}{C}; \quad (\text{A-34})$$

where $h_2 \equiv \frac{1-\lambda}{A_1} \left(\frac{A_2}{A_1} \frac{1}{1-\tau} \frac{1}{h} \right)^{-\varepsilon} + \frac{\lambda}{A_2} \left(\frac{1}{h_1} \right)^{-\varepsilon}$.

$$C_e = \nu p_e^{-\eta} C; \quad (\text{A-35})$$

$$C_g = (1 - \nu) p_g^{-\eta} C; \quad (\text{A-36})$$

$$e_g = \gamma D_g \left(\frac{p_e}{mc} \right)^{-\rho} C_g. \quad (\text{A-37})$$

$$E_1 = (1 - \lambda) \left(\frac{A_2}{A_1} \frac{1}{1-\tau} \frac{1}{h} \right)^{-\varepsilon} (C_e + e_g); \quad (\text{A-38})$$

$$E_2 = \lambda \left(\frac{1}{h_1} \right)^{-\varepsilon} (C_e + e_g); \quad (\text{A-39})$$

$$A_g = \bar{A}_g \exp(-\gamma E_1); \quad (\text{A-40})$$

$$A_1 = \bar{A}_1 \exp(-\gamma E_1); \quad (\text{A-41})$$

$$A_2 = \bar{A}_2 \exp(-\gamma E_1); \quad (\text{A-42})$$

$$N_{1,t} = \frac{U'(C) Y_g}{1 - \beta \theta \Pi^{-1} \Pi_g^\xi}; \quad (\text{A-43})$$

$$N_2 = \frac{U'(C) \frac{Mc}{P} Y_g}{1 - \beta \theta \Pi_g^\xi}; \quad (\text{A-44})$$

where we used the the market clearing condition that requires $Y_{g,t} = C_{g,t}$.

$$b = \frac{w_t + \hat{\pi} + \hat{T} - C}{1 - I/\Pi};$$

$$\hat{T} = \frac{A_2 P_e}{h} \frac{\tau}{1-\tau} + b \left(\frac{I}{\Pi} - 1 \right),$$

$$H_g = (1 - \gamma) \frac{Y_g D_g}{A_g} \left(\frac{w}{A_g mc} \right)^{-\rho},$$

$$H_1 = \frac{E_1}{A_1},$$

$$H_2 = \frac{E_2}{A_2}.$$

A.6 The model with flexible prices

With flexible prices, $D_{g,t} = 1, \forall t$. The first order condition w.r.t. H_t becomes

$$w_t = \psi H_t^\varphi C_t. \quad (\text{A-45})$$

Combining (A-45) with the market clearing condition for labor, (28), and using the definitions $\Theta_t \equiv (1 - \gamma) \left(\frac{1}{A_{g,t}}\right)^{1-\rho} + \gamma \left(\frac{h_t}{A_{2,t}}\right)^{1-\rho}$ and $h_{2,t} \equiv \frac{1-\lambda}{A_{1,t}} \left(\frac{A_{2,t}}{A_{1,t}} \frac{1}{1-\tau_t} \frac{1}{h_t}\right)^{-\varepsilon} + \frac{\lambda}{A_{2,t}} \left(\frac{1}{h_t}\right)^{-\varepsilon}$, we arrive at the following closed-form expression for C_t .

$$C_t = \left[\frac{\left[(1-\nu)\Theta_t^{\frac{1-\eta}{1-\rho}} + \nu h_t^{1-\eta} \right]^{\frac{1+\eta\varphi}{\varphi(\eta-1)}}}{\psi^{\frac{1}{\varphi}} \left((1-\gamma)(1-\nu)A_{g,t}^{\rho-1}\Theta_t^{\frac{\rho-\eta}{1-\rho}} + \left(\nu \left(\frac{h_t}{A_{2,t}}\right)^{-\eta} + (1-\nu)\gamma\Theta_t^{\frac{\rho-\eta}{1-\rho}} \left(\frac{h_t}{A_{2,t}}\right)^{-\rho} \right) h_{2,t} \right)} \right]^{\frac{\varphi}{1+\varphi}}. \quad (\text{A-46})$$

The prices are given by

$$mC_{g,t} = w_t \Theta_t^{\frac{1}{1-\rho}}, \quad (\text{A-47})$$

$$w_t = \left[(1-\nu) \Theta_t^{\frac{1-\eta}{1-\rho}} + \nu h_t^{1-\eta} \right]^{\frac{1}{\eta-1}}, \quad (\text{A-48})$$

$$p_{e,t} = w_t \frac{h_t}{A_{2,t}}, \quad (\text{A-49})$$

$$p_{g,t} = mC_{g,t},$$

$$\frac{mC_{g,t}}{w_t} = \Theta_t^{\frac{1}{1-\rho}},$$

$$\frac{mC_{g,t}}{p_{e,t}} = \frac{A_{2,t}\Theta_t^{\frac{1}{1-\rho}}}{h_t}.$$

Labor supply can then be solved from (A-45), and shown to be

$$H_t = \left[\frac{1}{\psi} \frac{w_t}{C_t} \right]^{\frac{1}{\varphi}}. \quad (\text{A-50})$$

With consumption and all prices known, it is straightforward to solve for the rest of the endogenous variables in the economy by using the expressions in Section 2.6. Specifically, $C_{g,t}$, $C_{e,t}$, $E_{1,t}$, $E_{2,t}$, and $e_{g,t}$ are respectively given by (17), (18), (21), (22), (24), whereas $H_{g,t}$, $H_{1,t}$, and $H_{2,t}$ can be directly solved for from (31)–(33).

The derivation of (38) makes use of combining the Fisher equation with the Taylor rule, i.e.,

$$R_t = \frac{\beta^{-1}\Pi_t^{\alpha\pi}}{\Pi_{t+1}},$$

A.7 Proof of Proposition 1

Proof. We have already established that without energy in production, $P_{g,t} = W_t$. Assume now, that these nominal prices are fixed at $P_{g,t} = W_t = \bar{W}_t$. From (36) it follows

that the nominal price of energy reads

$$P_{e,t} = \bar{W}_t \left[(1 - \lambda) \left(\frac{1}{1 - \tau_t} \right)^{1-\varepsilon} + \lambda \right]^{\frac{1}{1-\varepsilon}}. \quad (\text{A-51})$$

The CPI index reads

$$P_t = [(1 - \nu) P_{g,t}^{1-\eta} + \nu P_{e,t}^{1-\eta}]^{\frac{1}{1-\eta}}.$$

Inserting $P_{g,t} = \bar{W}_t$ and $P_{e,t} = \bar{W}_t \left[(1 - \lambda) \left(\frac{1}{1 - \tau_t} \right)^{1-\varepsilon} + \lambda \right]^{\frac{1}{1-\varepsilon}}$ into the CPI index delivers

$$P_t = \bar{W}_t \left[(1 - \nu) + \nu \left(\left[(1 - \lambda) \left(\frac{1}{1 - \tau_t} \right)^{1-\varepsilon} + \lambda \right]^{\frac{1}{1-\varepsilon}} \right)^{1-\eta} \right]^{\frac{1}{1-\eta}}$$

or

$$\frac{\bar{W}_t}{P_t} \equiv w_t = \left[(1 - \nu) + \nu \left(\left[(1 - \lambda) \left(\frac{1}{1 - \tau_t} \right)^{1-\varepsilon} + \lambda \right]^{\frac{1}{1-\varepsilon}} \right)^{1-\eta} \right]^{\frac{1}{\eta-1}},$$

which is exactly the same as (34). Hence, the increase in the CPI implements the efficient real wage. In addition, since $p_{g,t} = w_t$, equation (35) holds and, consequently, also the efficient goods price is implemented. \square

A.8 Proof of Proposition 2

Proof. Assume now that the carbon tax initially is τ_0 and that the nominal wage and goods price are fixed at their initial values, i.e.,

$$W_t = \bar{W}_t,$$

and

$$P_{g,t} = \bar{W}_t \left(1 - \gamma + \gamma \left[(1 - \lambda) \left(\frac{1}{1 - \tau_0} \right)^{1-\varepsilon} + \lambda \right]^{\frac{1-\rho}{1-\varepsilon}} \right)^{\frac{1}{1-\rho}}.$$

The tax is then changed to $\tau_t \neq \tau_0$, and the flexible nominal energy price becomes

$$P_{e,t} = \bar{W}_t \left[(1 - \lambda) \left(\frac{1}{1 - \tau_t} \right)^{1-\varepsilon} + \lambda \right]^{\frac{1}{1-\varepsilon}}.$$

Inserting these prices into the CPI index delivers, after some manipulation

$$\frac{\bar{W}_t}{P_t} = w_t = \left[(1 - \nu) \left(1 - \gamma + \gamma \left[(1 - \lambda) \left(\frac{1}{1 - \tau_0} \right)^{1 - \varepsilon} + \lambda \right]^{\frac{1 - \rho}{1 - \varepsilon}} \right)^{\frac{1 - \eta}{1 - \rho}} \right. \\ \left. + \nu \left[(1 - \lambda) \left(\frac{1}{1 - \tau_t} \right)^{1 - \varepsilon} + \lambda \right]^{\frac{1 - \eta}{1 - \varepsilon}} \right]^{\frac{1}{\eta - 1}} .$$

The above expression differs from (34) because $\tau_0 \neq \tau_t$. In particular, if $\tau_0 < \tau_t$ the CPI increases less than optimal (because $P_{g,t}$ cannot adjust), which results in a $p_{g,t}$ that is too low and w_t that is too high.

$$P_{g,t} = \bar{W}_t \left(1 - \gamma + \gamma \left[(1 - \lambda) \left(\frac{1}{1 - \tau_0} \right)^{1 - \varepsilon} + \lambda \right]^{\frac{1 - \rho}{1 - \varepsilon}} \right)^{\frac{1}{1 - \rho}}$$

□