MacroII, 2019-03-08
David Vestin

Assignment 1 - log-linearization, due date: Thursday 14/3.

1. Regular Taylor expansions: do the below exercise for two values of σ : 1.01 and 6. Compare results.

Let $f(x) = \frac{x^{1-\sigma}}{1-\sigma}$

a) Compute a first-order linear Taylor-expansion $f^1(x)$ to $f(x)$ around the point $x = 0.9$.

b) Compute $f^1(1.3)$ and the associated approximation error.

c) Redo a) but by computing the first-order approximation $f^2(x)$ around the point $x = 1.25$. Compare the approximation errors. Illustrate graphically. Conclusion?

d) The definition of relative risk-aversion is $R = -\frac{U''(C)C}{U'(C)}$. Compute R for the utility function $f(x)$ above.

2. Log-linearizations

Redo exercise 1, but log-linearize to a first-order in stead of linearize.

3. Consider the aggregate resource constraint

$$Y = C + I$$

where C is consumption and I is investment.

a) Linearize and write in difference from steady state

b) Log-linearize and write in log-difference from steady state

c) What is the difference in interpretation between $Y - \bar{Y}$ and $\log Y - \log \bar{Y}$?

4. Consider the intertemporal Euler-equation

$$C_t^{-\sigma} = \beta E_t \left(C_{t+1}^{-\sigma} r_t \right)$$

a) Assume that C_t and r_t (consumption and the real interest rate) are stationary variables and log-linearize around an assumed steady state.

b) Explain intuitively why the discount factor β is not part of the linearized equations.

5. Let the Euler-equation be

$$U'(C_t) = \beta E_t \left(U'(C_{t+1}) \frac{R_t}{\Pi_{t+1}} \right).$$

where R_t is the nominal interest rate and Π_t is gross inflation (P_t/P_{t-1}). Log-linearize this equation to a first order and describe the only feature of the utility function that will have an impact on the approximate equation.

6. Assume that

$$
\begin{aligned}
U\left(C_t, N_t\right) &= \log\left(C_t\right) - \gamma \frac{N_t^{1+\phi}}{1+\phi} \\
P_t C_t + B_t &= W_t N_t + \left(1 + r_{t-1}\right) B_{t-1} + D_t
\end{aligned}
$$

and that consumers maximize the expected discounted stream of current and future utility, where B_t is the amount invested in bonds at time t that will mature in $t+1$ and D_t is dividens from firms.

Assume perfect competition both at the prodcut and labour markets and let firms hire workers at the nominal wage W_t. Let the production function be

$$
\begin{aligned}
Y_t &= A_t H_t^{\alpha} \\
\log A_t &= \rho \log A_{t-1} + \varepsilon_t
\end{aligned}
$$

where ε_t is iid normal with variance σ^2, and where $\alpha < 1$.

a) Set up the consumer problem and take first-order conditions.

b) Maximize profits for firms

c) Solve for the steady state

d) Try to solve the model analytically. Hint:

e) Log-linearize the model and solve for the approximate real equilibrium.

f) Calculate the effect of a technology shock on output, real wages and employment. Discuss how various parameters affect the solution.

g) Assume that $\rho = 1$. What problem does this introduce for the log-linearization (same as for linearization)? Show how to deal with it and redo the computation.