MakroII, Assignment 2: The NK Phillip's curve

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Due: 2019-03-21, before Markus class.

1. Assume a utility function $U(C,N) = \frac{C^{1-\sigma}}{1-\sigma} - \gamma \frac{N^{1+\phi}}{1+\phi}$ where

$$C = \left( \int_0^1 c(i)^{\frac{\varepsilon-1}{\varepsilon}} \, di \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

where $i$ indexes a contiunuum of goods. Each good is produced by one firm.

a) Explain intuitively why this type of preferences introduces monopolistic competition, and why we need this assumption (or a similar one) if we add sticky prices to the model.

b) Assuming that consumer have chosen to consume $C$, how much will they consume of good $i$, if the price of that good is $p(i)$? That is, derive demand functions $c_t(i)$. As part of the solution, derive an expression for the aggregate price-level $P_t$.

c) Compute $\frac{\partial c(i)}{\partial p(i)}$ and discuss how this expression is affected by the parameter $\varepsilon$.

d) Compute the elasticity of substitution between two goods.

e) Explain why the pareto-allocation involves $c(i) = c(j)$.

f) Explain why the condition in e) is typically violated if we assume that prices are sticky.

g) What does the parameter $\phi$ intend to capture?

2. Given the utility function in 1, assume that the representative agent can invest in a one-period bond $B_t$ and that the risk-free gross nominal interest rate is $I_t$, and that the common wage in the economy is $W_t$. Set up the individuals intertemporal optimization problem and take first order conditions. Discuss how various parameters affect the labor supply condition (the combined first-order conditions to consumption and hours worked) and the Euler-equation.

3. Expanding on the model given in 1 and 2, sssume that firms produce their goods with the production function$Y_t(i) = A_t N_t(i)^{1-\alpha}$ where $\log A_t = \rho \log A_{t-1} + \varepsilon_t^a$

a) Assume that prices are flexible, set up and solve the profit maximizing problem of the firm, under the assumption that it faces a given wage rate $W_t$.

b) Assume the limiting case $\sigma -> 1$ and $\alpha = 0$. Set up the profit maximization problem of firms under the assumption that only a fraction $\omega$ of firms may change their prices optimally in every period, and that remaining firms keep their prices unchanged, and take first-order conditions.

c) Prove that in equilibrium, the log-linearized first-order conditions in b) above can be written as,

$$\pi_t = \beta E_t \pi_{t+1} + \lambda mc_t$$

and find the value of $\lambda$ expressed in terms of the structural parameters of the model.

d) Discuss how an increase in $\gamma$ affects the soultion to the model.

e) Explain how we can translate $mc_t$ to output

f) Show how we can combine the results above to arrive at

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t$$

where $x_t = \log y_t - \log y_t^n$.

g) Compute the natural rate of interest $r_t^n$ in this economy and discuss how it is affected by a technology shock. Discuss how monetary policy should optimally respond.