MakroII, Assignment 2: The NK Phillip's curve

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Due: 2020-05-25, before Markus class.

1. Assume a utility function $U(C,N) = \frac{C^{1-\sigma}}{1-\sigma} - \frac{N^{1+\phi}}{1+\phi}$ where

$$C = \left(\int_0^1 c(i)^{\frac{\varepsilon-1}{\varepsilon}} \, di \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

where i indexes a contiunuum of goods. Each good is produced by one firm.

a) Explain intuitively why this type of preferences introduces monopolistic competition, and why we need this assumption (or a similar one) if we add sticky prices to the model.

b) Assuming that consumer have chosen to consume C, how much will they consume of good i, if the price of that good is $p(i)$? That is, derive demand functions $c_t(i)$. As part of the solution, derive an expression for the aggregate price-level P_t.

c) Explain why the pareto-allocation involves $c(i) = c(j)$.

2. Assume that firms produce their goods with the production function$Y_t(i) = A_t N_t(i)^{1-\alpha}$ where $\log A_t = \rho \log A_{t-1} + \varepsilon_t^a$ and that consumers are as in 1 above.

a) Assume that prices are flexible, set up and solve the profit maximizing problem of the firm, under the assumption that it faces a given wage rate W_t.

b) Assume the limiting case $\sigma -> 1$ and $\alpha = 0$. Now assume Calvo pricing, assuming that only a fraction ω can change their prices every period. Set up the profit maximization problem and take first-order conditions.

c) Prove that in equilibrium, the log-linearized first-order conditions in b) above can be written as,

$$\pi_t = \beta E_t \pi_{t+1} + \lambda mc_t$$

and find the value of λ expressed in terms of the structural parameters of the model.

d) Find the flex-price output level and discuss the link between marginal cost and the output-gap.

3. Now consider the full model

$$\begin{aligned} \pi_t &= \beta E_t \pi_{t+1} + \kappa x_t \\ x_t &= E_t x_{t+1} - \sigma^{-1} \left(i_t - E_t \pi_{t+1} - r_t^n \right) \\ r_t^n &= \rho - \psi a_t \end{aligned}$$

where $\psi > 0$ and $x_t = \log y_t - \log y_t^n$.

a) Add a Taylor-rule $i_t = \rho + \alpha \pi_t + v_t$ and solve the model. Discuss what happens in the first period when a monetary policy shock hits the economy. Next, discuss what happens in the first period when a technology shock hits.

b) Instead of the Taylor rule, assume that monetary policy is conducted by minimizing $L_t = \pi_t^2 + \gamma x_t^2$ under discretion. Solve the model under this assumption (notice that of course there is now no monetary policy shock) and compare with a).