Assignment 1 - log-linearization, due date: Monday 18/5, handed in by email before class 13.15.

1. The utility function $U(c) = \frac{c^{1-\gamma}}{1-\gamma}$ is standard in much of the macro-lit we will examine.
   (a) Compute the elasticity of substitution.
   (b) Compute the relative risk-aversion (definition of relative risk-aversion is $R = \frac{U''(C)C}{U'(C)}$ Compute R for the utility function U(C) above.). Compare to (a), conclusion?
   (c) Discuss why the functional form is convenient (and hopefully a good approximation of the truth!) when there is growth in the economy.

2. Regular Taylor expansions: do the below exercise for two values of $\gamma : 1.01$ and 6. Compare results. Let $U(c) = \frac{c^{1-\gamma}}{1-\gamma}$
   (a) Compute the marginal utility of consumption.
   (b) Compute a Taylor-expansion of the marginal utility around the point $c = 0.9$.
   (c) Compute the true marginal utility when consumption is 1, that is $\frac{dU}{dx}(1)$. Compare this with your Taylor-expansion at the same point and compute the approximation error.
   (d) Redo (c) but now when $c = 1.1$. Conclusion?
   (e) Repeat $(b) - (d)$ but instead linearize (Taking a first-order Taylor expansion) around the point c = 1.05. Conclusion?

3. Log-linearizations
   (a) Log-linearize (instead of linearize) the marginal utility of consumption from $2(a)$ to a first-order around the point $c = 0.9$.
   (b) Compare the true marginal utility when consumption is 1 from $2(c)$ with your log linear approximation at the same point and compute the approximation error.
   (c) Redo (b) but now when $c = 1.1$. Conclusion?
   (d) Repeat $(a) - (c)$ but instead log-linearize around the point c = 1.05. Conclusion?

4. Consider the aggregate resource constraint

$$Y = C + I + G$$

   where C is consumption and I is investmentment and G is government consumption.
   (a) Linearize and write in difference from steady state.

(b) Log-linearize and write in log-difference from steady state.

(c) What is the difference in interpretation between $Y - \bar{Y}$ and $\log Y - \log \bar{Y}$?

5. Consider the inter-temporal Euler-equation

$$C_t^{-\sigma} = \beta E_t \left(C_{t+1}^{-\sigma} r_t \right)$$

   (a) Assume that C_t is stationary (no-growth). Find the steady state for the real interest rate.

   (b) Log-linearize around an assumed steady state.

   (c) Explain intuitively why the discount factor β is not part of the log-linearized equations.

6. Let the Euler-equation be

$$U'(C_t) = \beta E_t \left(U'(C_{t+1}) \frac{R_t}{\Pi_{t+1}} \right).$$

   where R_t is the nominal interest rate and Π_t is gross inflation (P_t/P_{t-1}). Log-linearize this equation to a first order and describe the only feature of the utility function that will have an impact on the approximate equation.

7. Assume that

$$\begin{aligned} U(C_t, N_t) &= \frac{C_t^{1-\gamma}}{1-\gamma} - \frac{N_t^{1+\phi}}{1+\phi} \\ P_t C_t + B_t &= W_t N_t + (1 + r_{t-1}) B_{t-1} + D_t \end{aligned}$$

   and that consumers maximize the expected discounted stream of current and future utility, where B_t is the amount invested in bonds at time t that will mature in $t+1$ and D_t is dividends from firms.

   Assume perfect competition both at the product and labour markets and let firms hire workers at the nominal wage W_t. Let the production function be

$$\begin{aligned} Y_t &= A_t H_t^{\alpha} \\ \log A_t &= \rho \log A_{t-1} + \varepsilon_t \end{aligned}$$

   where ε_t is iid normal with variance σ^2, and where $\alpha < 1$.

   (a) Set up the intertemporal consumer problem and take first-order conditions.

   (b) Maximize profits for firms.

(c) Solve for the steady state.

(d) Try to solve the model analytically. Hint: it is not too hard to compute the expected value of a variable that has a log-normal distribution.

(e) Log-linearize the model and solve for the approximate real equilibrium.

(f) Calculate the effect of a technology shock on output, real wages and employment. Discuss how various parameters affect the solution.

(g) Assume that $\rho = 1$. What problem does this introduce for the log-linearization (same as for linearization)? Show how to deal with it and redo the computation.